CS21004 Formal Languages and Automata Theory, Spring 2012–13

Class test 2

Maximum marks: 20 | Date: 11-April-2013 | Duration: 1 hour

Roll no: _____ **Name:** _____

[*Write your answers in the question paper itself. Be brief and precise. Answer all questions.*]

1. (a) Design a PDA with *only one state* to accept the following language: **(8)**

$$L_1 = \{\alpha \in \{a, b\}^* \mid \#a(\alpha) \leqslant \#b(\alpha)\}$$

Solution Let $\alpha \in L_1$ contain m a's. The number of b's in α would be $m + n$ for some $n \geqslant 0$. The PDA matches m a's against m b's, and ignores the remaining n b's. However, m, n are not known a priori, so the machines guesses them non-deterministically. This means that it makes a non-deterministic decision whether some b is to be matched against an a or it would be ignored. Since the PDA has only one state, it would accept by empty stack. We will let S denote the bottom marker for the stack. When the entire string is read from the input, the bottom marker should be exposed. It is then popped out, and the machine accepts. We use two other stack symbols A and B to indicate the excess of a's and b's (respectively) read so far. The PDA has the following transitions:

$a, S/AS$	(All a's must be matched, so push A's)
$a, A/AA$	(whenever there are not surplus b's so far)
$a, B/\epsilon$	(One a matched with an earlier b)
$b, S/BS$	(This b is to be matched with an a)
$b, B/BB$	(This b is to be matched)
$b, A/\epsilon$	(This b is to be matched)
$b, S/S$	(This b is not matched)
$b, B/B$	(This b is not matched)
$b, A/A$	(This b is not matched)
$\epsilon, S/\epsilon$	(The last move to empty the stack)

(b) Convert the PDA of Part (a) to an equivalent CFG. **(4)**

Solution The above transitions can be mechanically converted to the following productions. Here, S is the start symbol.

$$\begin{aligned} S &\rightarrow aAS \mid bBS \mid bS \mid \epsilon \\ A &\rightarrow aAA \mid b \mid bA \\ B &\rightarrow a \mid bBB \mid aB \end{aligned}$$

2. Design a total Turing machine to accept the following language:

$$L_2 = \{\alpha \in \{a, b, c\}^* \mid \#a(\alpha) \leqslant \#b(\alpha) \leqslant \#c(\alpha)\}$$

Briefly describe the working of the machine (how the head moves and rewrites tape cells). You may use the example $cbcabbcac$ for illustration. **(8)**

Solution The Turing machine M that we are going to design makes a to-and-fro motion along the tape, and erases matching a's with b's and c's, and then erases matching b's with c's. First, the machine puts a right end-marker $\dashv$ at the end of the input. It then goes to the left end-marker $\vdash$, and makes a first forward pass. It then moves forward until it finds an a or encounters $\dashv$. If an a is found, the head moves back to the leftmost cell, and in one (or two) forward movements attempts to erase one b and one c. If the attempt is unsuccessful, there are more a's than b's or c's, so M rejects (and halts). If all a's are successfully erased, then a pass is used to erase one b followed by another pass for erasing one c. If at some point a b is found but no c is left, the input contains more b's than c's. M rejects and halts in that case. However, if all b's are successfully erased, M accepts and halts. The tape content after the different passes for the input $cbcabbcac$ is shown below.

Initially	$\vdash$	c	b	c	a	b	b	c	a	c	$\sqcup$	All the remaining tape cells are blank
After Pass 1	$\vdash$	c	b	c	a	b	b	c	a	c	$\dashv$	The right end-marker is placed
After Pass 2	$\vdash$	c	b	c	$\sqcup$	b	b	c	a	c	$\dashv$	One a is erased
After Pass 3	$\vdash$	$\sqcup$	$\sqcup$	c	$\sqcup$	b	b	c	a	c	$\dashv$	One b and one c are erased
After Pass 4	$\vdash$	$\sqcup$	$\sqcup$	c	$\sqcup$	b	b	c	$\sqcup$	c	$\dashv$	One a is erased
After Pass 5	$\vdash$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	b	c	$\sqcup$	c	$\dashv$	One b and one c are erased
After Pass 6	$\vdash$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	b	c	$\sqcup$	c	$\dashv$	No more a found
After Pass 7	$\vdash$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	c	$\sqcup$	c	$\dashv$	One b erased
After Pass 7	$\vdash$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	c	$\dashv$	One c erased
After Pass 7	$\vdash$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	c	$\dashv$	No b found. Accept and halt.