CS21004 Formal Languages and Automata Theory, Spring 2012–13

Class test 1

Maximum marks: 20 | Date: 13-February-2013 | Duration: 1 hour

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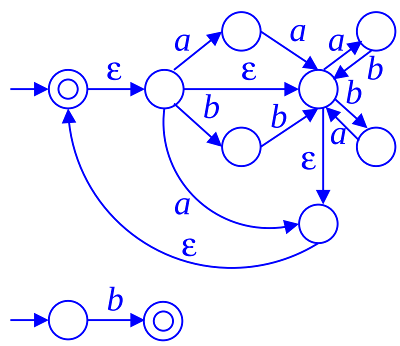
Roll no: ____________________ **Name:** ____

[*Write your answers in the question paper itself. Be brief and precise. Answer all questions.*]

1. Construct an ϵ-NFA equivalent to the regular expression $((aa+bb+\epsilon)(ab+ba)^*+a)^*+b$. **(6)**

Solution



2. (a) Give an example of a non-regular language L for which the asterate L^* is regular. **(4)**

Solution Take

$$L = \{a^p \mid p \text{ is a prime }\}.$$

It is easy to see that

$$L^* = \{a^n \mid n \neq 1\} = \mathcal{L}(\epsilon + aaa^*).$$

(b) Suppose that L_1 and L_2 are two languages (over the same alphabet) given to you such that both L_1 and L_1L_2 are regular. Prove or disprove: L_2 must be regular too. **(4)**

Solution This is false. For example, take

$$L_1 = \mathcal{L}(a^*),$$

and

$$L_2 = \{a^p \mid p \text{ is a prime}\}.$$

But then,

$$L_1L_2 = \{a^n \mid n \geqslant 2\} = \mathcal{L}(aaa^*).$$

3. Using the pumping lemma, prove that the language $L_3 = \{a^i b^j \mid i, j \geqslant 0, \text{ and } |i - j| \text{ is a prime}\}$ is not regular. (Note that 1 is not treated as a prime.) **(6)**

Solution Suppose that L_3 is regular. Let k be a pumping-lemma constant for L_3. Feed the string $\alpha\beta\gamma = a^{k+2}b^k$ with $\alpha = \epsilon$, $\beta = a^{k+2}$ and $\gamma = b^k$, to the pumping lemma. We get a decomposition $\beta = \beta_1\beta_2\beta_3$ with $l = |\beta_2| \geqslant 1$. Now, take $i = 3$, that is, pump in β_2 twice in $\alpha\beta\gamma$ to get the string $\alpha\beta_1\beta_2^3\beta_3\gamma = a^{k+2+2l}b^k \in L_3$. This is a contradiction, since $2 + 2l = 2(1 + l)$ is not a prime.