CS21004 Formal Languages and Automata Theory, Spring 2010–11

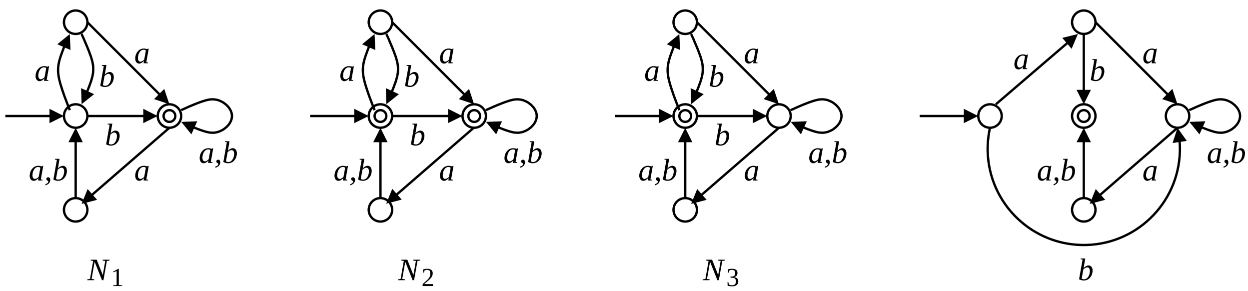
Mid-Semester Test

Maximum marks: 30 | Date: February 2011 | Duration: 2 hours

Roll no: ____________ **Name:** ______________________________

[*Write your answers in the question paper itself. Be brief and precise. Answer all questions.*]

1. From the following four NFAs and four regular expressions, identify the equivalent pairs. **(6)**



N_1 N_2 N_3 N_4

$$\alpha_1 = ab + (aa+b)(a+b)^*a(a+b)$$
$$\alpha_2 = (ab + (aa+b)(a+b)^*a(a+b))^*$$
$$\alpha_3 = (ab + (aa+b)(a+b)^*a(a+b))^*(aa+b)(a+b)^*$$
$$\alpha_4 = (ab + (aa+b)(a+b)^*a(a+b))^*(\epsilon + (aa+b)(a+b)^*)$$

The NFA N_1 is equivalent to the regular expression ____ α_3 ____ .

The NFA N_2 is equivalent to the regular expression ____ α_4 ____ .

The NFA N_3 is equivalent to the regular expression ____ α_2 ____ .

The NFA N_4 is equivalent to the regular expression ____ α_1 ____ .

2. Consider the following four context-free grammars over the alphabet $\{a, b\}$ and with the start symbol S.

$S \to aaS \mid Sbb \mid \epsilon$	$S \to aaSSb \mid aSSbb \mid \epsilon$	$S \to aSb \mid bSa \mid \epsilon$	$S \to aSb \mid bSa \mid a \mid b$
Grammar G_1	Grammar G_2	Grammar G_3	Grammar G_4

For each of the following strings, there is a unique grammar among the above four, which generates the string. Identify these respective grammars. **(6)**

The string $aabbaabb$ is generated by the grammar ____ G_3 ____ .

The string $aabbbbbb$ is generated by the grammar ____ G_1 ____ .

The string $aabbbaabb$ is generated by the grammar ____ G_4 ____ .

The string $aabbaabbb$ is generated by the grammar ____ G_2 ____ .

3. Let L_1 and L_2 be regular languages over the alphabet Σ. Define the language

$$L_3 = \{\alpha\beta\gamma \mid \alpha\gamma \in L_1,\ \beta \in L_2\},$$

that is, L_3 is obtained by inserting strings in L_2 inside strings in L_1. Prove that L_3 is regular too. **(6)**

Solution Let $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ be DFA with languages L_1 and L_2, respectively. Design an ϵ-NFA $M_3 = (Q_3, \Sigma, \Delta_3, S_3, F_3)$ as follows. M_3 contains two copies of M_1, and n copies of M_2, where $n = |Q_1|$ is the number of states of M_1. Let $Q_1 = \{q_1, q_2, \ldots, q_n\}$. The i-th copy of M_2 in M_3 is meant for making a detour from M_1 at the state q_i, follow the transitions in the i-th copy of M_2 labelled by a string in L_2, and return back to q_i. This means that from q_i in the first copy of M_1, we add an ϵ-transition to the start state of the i-th copy of M_2. In addition, we add an ϵ-transition from each final state of the i-th copy of M_2 to the state q_i in the second copy of M_1. M_3 has a unique start state, namely, the start state s_1 of the first copy of M_1. The final states of M_3 are precisely all the final states in the second copy of M_1.

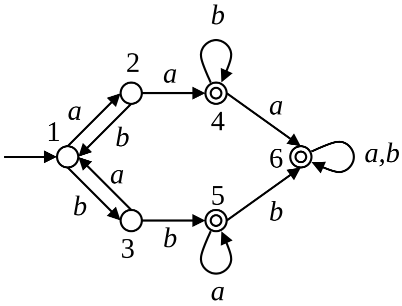
4. Consider the following language over $\Sigma = \{a, b, c\}$:

$$L = \{xcy \mid x, y \in \{a, b\}^*,\ \#a(x) = \#b(y)\}.$$

Using the pumping lemma, prove that L is not regular. **(6)**

Solution Suppose that L is regular. Let n be a pumping-lemma constant for L. Take the string $\alpha\beta\gamma = a^n cb^n \in L$, where $\alpha = a^n c$, $\beta = b^n$ and $\gamma = \epsilon$. By the pumping lemma, we have a decomposition $\beta = \beta_1\beta_2\beta_3$ with $k = |\beta_2| \geqslant 1$ and $\alpha\beta_1\beta_3 = a^n cb^{n-k} \in L$. But $\beta_1\beta_3 = b^{n-k}$ contains less number of b's than the number of a's in a^n, a contradiction.

5. Consider the following DFA.



Use the DFA state-minimization procedure to convert this DFA to an equivalent DFA with the minimum possible number of states. Also draw the quotient automaton. **(6)**

Solution Initialization:

	1	2	3	4	5	6
1	–					
2	–	–				
3	–	–	–			
4	–	–	–	–		
5	–	–	–	–	–	
6	–	–	–	–	–	–

Initial conflicts:

	1	2	3	4	5	6
1	–					
2	–	–				
3	–	–	–			
4	×	×	×	–		
5	×	×	×	–	–	
6	×	×	×	–	–	–

Pass 1:

	1	2	3	4	5	6
1	–					
2	×	–				
3	×	×	–			
4	×	×	×	–		
5	×	×	×	–	–	
6	×	×	×	–	–	–

No further conflicts can be found. We see that the states $4, 5, 6$ are equivalent. Therefore, we collapse these three states into one, and obtain the quotient automaton as follows: