CS21004 Formal Languages and Automata Theory, Spring 2010–11

Class test 1

Maximum marks: 20 Date: February 10, 2011 Duration: 1 hour

Roll no: _____ **Name:** _____

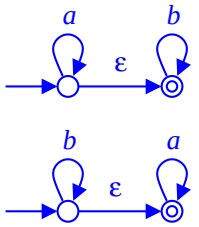
[*Write your answers in the question paper itself. Be brief and precise. Answer all questions.*]

1. Let L_1 be the language of the regular expression $a^*b^* + b^*a^*$.

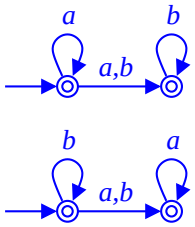
(a) Give an example of a string $\{a, b\}^*$ which is not in L_1. *aba* **(1)**

(b) Design an NFA with four states to accept L_1. You may use ϵ-transitions. **(5)**

Solution



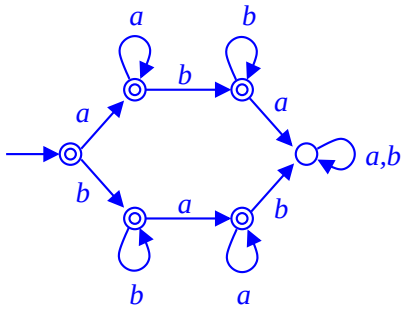
(i) An NFA with epsilon transitions



(ii) An NFA without epsilon transitions

(c) Design a DFA with six states to accept L_1. **(5)**

Solution

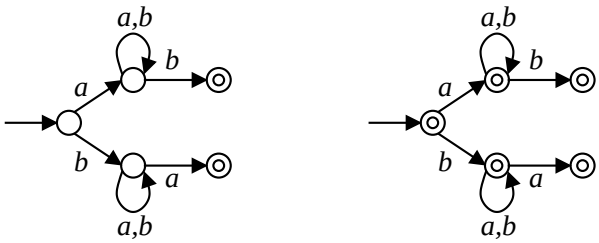


2. A string $\beta \in \Sigma^*$ is called a *prefix* of a string $\alpha \in \Sigma^*$ if $\alpha = \beta\gamma$ for some $\gamma \in \Sigma^*$. For example, all the prefixes of $abaa$ are $\epsilon, a, ab, aba, abaa$. Let $L \subseteq \Sigma^*$ be a language. By $\text{prefix}(L)$, we denote the set of all prefixes of all strings in L.

(a) If $L = \{aab, bab, abab\}$, what is $\text{prefix}(L)$?

$\{\epsilon, a, b, aa, ab, ba, aab, aba, bab, abab\}$ **(1)**

(b) Let L_2 be the language over $\Sigma = \{a, b\}$, accepted by the NFA of Part (i) in the following figure.



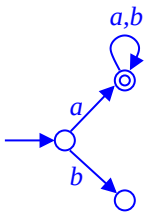
L_2 consists of strings that start and end with different symbols. **(1)**

$\text{prefix}(L_2)$ consists of all strings in $\{a, b\}^*$. **(1)**

(c) The NFA of Part (ii) in the above figure is obtained by converting each state of the NFA of Part (i) to a final state. State whether the converted NFA accepts $\text{prefix}(L_2)$. Yes (Write Yes/No) **(1)**

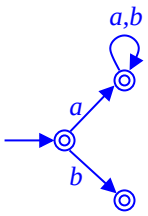
(d) Let $N = (Q, \Sigma, \Delta, S, F)$ be an NFA, and $N' = (Q, \Sigma, \Delta, S, Q)$ be the NFA obtained from N by converting every state of N to a final state. Prove or disprove: We must have $\mathcal{L}(N') = \text{prefix}(\mathcal{L}(N))$. **(5)**

Solution The statement is false. A counterexample is given below.



This NFA accepts all strings starting with *a*

The prefix of this language is the set of all strings starting with *a* and the empty string



This NFA accepts *b* which is not in the prefix language

Remark: The corrected construction goes like this: Convert a state $q \in Q$ of N to a final state if and only if there is a path from q to any final state of N, that is, if and only if $\hat{\Delta}(q, \alpha) \cap F \neq \emptyset$ for some $\alpha \in \Sigma^*$.