End Semester Examination, Autumn 2003-04

Total points: 100 | November 25, 2003 | Total time: 3 hours

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1. Which of the following languages is/are context-free? (Give sufficient justifications.) **(5×5)**

(a) $\{a^i b^j c^k \mid i, j, k \geqslant 0 \text{ and } i + j = k\}$.

(b) The set of all concatenations of strings of the form $a^i b^j c^k$ with $i, j, k \geqslant 0$ and $i + j = k$, i.e., the language $\{a^{i_1} b^{j_1} c^{k_1} a^{i_2} b^{j_2} c^{k_2} \ldots a^{i_l} b^{j_l} c^{k_l} \mid l, i_r, j_r, k_r \geqslant 0 \text{ and } i_r + j_r = k_r \text{ for all } r = 1, 2, \ldots, l\}$.

(c) $\{(a^i b^j c^k)^l \mid i, j, k, l \geqslant 0 \text{ and } i + j = k\}$.

(d) $\{a^{il} b^{jl} c^{kl} \mid i, j, k, l \geqslant 0 \text{ and } i + j = k\}$.

(e) $\{a^{i+l} b^{j+l} c^{k+l} \mid i, j, k, l \geqslant 0 \text{ and } i + j = k\}$.

2. Consider the CFL $L := \{a^k b^l \mid l \geqslant 0 \text{ and } 0 \leqslant k \leqslant 2l\}$.

(a) We have discussed the following CFG for L:

$$S \to \epsilon \mid Sb \mid aSb \mid aaSb\,.$$

Show that this grammar is ambiguous. **(5)**

(b) Design an unambiguous CFG for L. You must supply a proof that your grammar is unambiguous. **(15)**

3. An arithmetic expression like `((a+b)*(c-d)/e)` is said to be in the infix notation, since each operator is placed between its operands. In the postfix notation the operator follows its two operands. Thus `a+b` and `c-d` in the postfix notation become `ab+` and `cd-` and `(((a+b)*(c-d))/e)` becomes `ab+cd-*e/`. It can be proved that the postfix notation does not require parentheses to resolve ambiguities.

(a) Write the postfix expression for $a + \cfrac{1}{b + \cfrac{1}{a + \cfrac{1}{b}}}$ (i.e., for `(a+(1/(b+(1/(a+(1/b))))))`). Also convert the postfix expression `abab+/ab-*a/*b-` to the equivalent infix expression. **(5)**

(b) Design a PDA whose language is the set of all valid postfix expressions involving two variables `a` and `b` and the binary operators `+`, `-`, `*` and `/`. **(10)**

4. **(a)** Describe a one-tape one-head TM whose language is the set of all strings over $\{a, b\}$, that contain equal number of a and b. What is the maximum number of moves this TM requires for a string of length n? **(10)**

(b) Show that using a scratch tape one can reduce this maximum number of moves. **(10)**

5. **(a)** Let L be an R.E. language over some alphabet Σ. Show that L is the language of some TM that does not halt on some input(s), if and only if $L \neq \Sigma^*$. **(10)**

(b) Show that R.E. languages are closed under union and intersection. **(10)**

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