CS21001 Discrete Structures, Autumn 2007

Class test 2: November 07, 2007 (7:00–8:00pm), Total Marks: 30

Roll No: ____________________ **Name:** ____

[*Write your answers in the respective spaces provided in the question paper itself. Answer briefly.*]

1 On two sides of a battle-field, there are two camps of soldiers. Initially, Camp 1 consists of n_1 soldiers, and Camp 2 of n_2 soldiers. Each camp sends one soldier to the battle-field. The two soldiers fight for one minute and exactly one of them dies. The living soldiers rejoins his camp. This process continues as long as each camp has (at least one) soldier to continue the battle. In terms of a C program, the battle looks as follows.

```
int battle ( unsigned int n1, unsigned int n2 )
{
    int t = 0, whowins;
    while ((n1 > 0) && (n2 > 0)) {          /* as long as both camps are non-empty */
        whowins = findWinner();                      /* findWinner() returns 1 or 2 */
        if (whowins == 1) --n2; else --n1;              /* the losing soldier dies */
        ++t;                                          /* increase time by 1 minute */
    }
    return t;                            /* return the total duration of the battle */
}
```

(a) Let m be the minimum possible duration (in minutes) of the battle, i.e., the minimum value returned by the function `battle(n1,n2)`. Determine m (in terms of n_1, n_2). Justify your answer (i.e., argue that no battle can last shorter than m minutes and that a battle can last for m minutes.) **(5)**

Thus, $m =$ ____ .

(b) Let M be the maximum possible duration (in minutes) of the battle. Determine M with justification. **(5)**

Thus, $M =$ ________________________________ .

(c) Now imagine a modified situation in which two soldiers fight for one minute, but nobody dies. In that case, the two soldiers stop the fight and decide to go together to one of the two camps. This means that one of the two soldiers defects from his own camp and joins the other camp. This defection operation takes an additional minute (after the one-minute fight). Thus, each fight now has three outcomes: (a) the soldier from Camp 1 dies, (b) the soldier from Camp 2 dies, and (3) one of the two soldiers defect. The first two outcomes take 1 minute each, whereas the third outcome takes 2 minutes. Assume that if a soldier defects to the rival camp, he can never go back to his original camp.

Let M' be the maximum possible duration (in minutes) of the battle in this modified scenario. Determine M' with justification. You may assume that $n_1, n_2 \geqslant 2$. **(5)**

Thus, $M' =$ ________________________________ .

2 Let the sequence $a_0, a_1, a_2, \ldots$ be defined recursively as follows.

$$\begin{aligned} a_0 &= 0, \\ a_n &= a_{n-1} + a_{n-2} + \cdots + a_1 + a_0 + 2^{n+1} \quad \text{for } n \geqslant 1. \end{aligned}$$

Deduce a closed-form formula for a_n for all $n \geqslant 0$. You may use any method that you find convenient. You may use the back of this page, if your calculations do not fit in this page. **(15)**