CS21001 Discrete Structures, Autumn 2007

Class test 1

Total marks: 50 September 06, 2007 (6:00-7:00pm) **Duration:** 1 hour

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Roll No: ____________________ **Name:** ____

[*Write your answers in the respective spaces provided in the question paper itself. Answer briefly.*]

1 Prove or disprove the following assertions. **(5 × 5)**

(a) Let p, q, r, s be atomic propositions. Then, the compound proposition $p \wedge r \rightarrow q \vee \neg s$ is logically equivalent to the compound proposition $p \wedge s \rightarrow q \vee \neg r$.

(b) Let $P(x), Q(x)$ be predicates. Then the proposition $(\exists x)[P(x)] \rightarrow (\exists y)[Q(y)]$ is logically equivalent to $(\exists x)(\exists y)[P(x) \rightarrow Q(y)]$.

(c) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions such that $g \circ f : A \rightarrow C$ is a bijection. Then f must be a bijection too.

(d) The antisymmetric closure of a relation ρ on a set A exists if and only if ρ itself is antisymmetric.

(e) There exists a relation that well-orders $\mathbb{Z}$.

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2 Let F_n, $n \in \mathbb{N}$, denote the sequence of Fibonacci numbers. Prove by induction on n that **(15)**

$$F_{2n} = F_n(F_{n+1} + F_{n-1}) \quad \text{and} \quad F_{2n+1} = F_{n+1}^2 + F_n^2 \quad \text{for all } \; n \geqslant 1.$$

3 Let $f : \mathbb{N} \to \mathbb{N}$ be a bijection not equal to the identity map. Prove that there exists $n \in \mathbb{N}$ such that $n < f(n)$ and $n < f^{-1}(n)$. **(10)**