CS21001 Discrete Structures, Autumn 2006

Class test 1

Total marks: 30 September 12, 2006 (6:00-7:00pm) **Duration:** 1 hour

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1. Let $P(x), Q(x)$ be predicates involving an integer-valued variable x. Prove or disprove: $\forall x\,[P(x) \Rightarrow Q(x)]$ is logically equivalent to $\forall x\,[P(x)] \Rightarrow \forall x\,[Q(x)]$. **(5)**

2. The following recursive function takes as argument an array A of integers and its size $n \geqslant 1$.

```
int f ( int A[], unsigned int n )
{
   if (n == 1) return 0;
   if (n == 2) return A[1];
   return f(A,2) + f(&A[1],n-1) + f(&A[2],n-2);
}
```

(a) Let the element at index i in the array A be denoted by a_i. Prove by induction on n that the function returns $F_0a_0 + F_1a_1 + F_2a_2 + \cdots + F_{n-1}a_{n-1}$ for all $n \geqslant 1$, where F_i is the i-th Fibonacci number. **(10)**

(b) Let T_n denote the running time of the above function on an array of size n. Write a recurrence relation for T_n. Also supply the requisite number of initial conditions. **(5)**

(c) Solve the above recurrence relation to obtain an explicit formula for T_n. Conclude that $T_n = \Theta(\phi^n)$, where ϕ is the golden ratio. **(10)**