CS21001 Discrete Structures, Autumn 2005

Class test 2

Total marks: 20 | November 10, 2005 | **Duration:** $1 + \epsilon$ hour

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Roll No: ____________ **Name:** ____________________________

Answer all questions in the respective spaces provided.
Use extra sheets for rough work. Any such extra sheet will not be corrected.

1. Which of the following assertions is/are true. Give short justifications. No credits will be given without proper reasoning. **(2×5)**

(a) The set of all complex numbers of the form $x + \mathrm{i}y$ with x, y integers and with x even is a group under addition of complex numbers.

(b) Let G be a multiplicative group in which $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$. Then G is Abelian.

(c) Let $f : G_1 \to G_2$ be a homomorphism of finite groups and $a \in G_1$. Then $\operatorname{ord} f(a)$ is an integral multiple of $\operatorname{ord} a$.

(d) Let G be a group and $m, n \in \mathbb{N}$ with $\gcd(m, n) = 1$. Assume that G contains elements a, b with $\operatorname{ord} a = m$ and $\operatorname{ord} b = n$. Then G is cyclic.

(e) Let H, K be subgroups of a finite multiplicative group G with $K \subseteq H$. Then $[G:K] = [G:H][H:K]$.

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2. Let G be a multiplicative group and H, K subgroups of G with $H \cap K = \{e\}$. Assume that $G = HK = \{hk \mid h \in H, k \in K\}$. Prove that every element $a \in G$ can be written as $a = hk$ for some *unique* elements $h \in H$ and $k \in K$. (Note: In this case G is called the *internal direct product* of H and K.) **(5)**

3. Prove that an infinite group has infinitely many distinct subgroups. **(5)**