CS21001 Discrete Structures, Autumn 2005

Class test 1

Total marks: 25 | September 08, 2005 | **Duration:** $1 + \epsilon$ hour

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Name: ______________________________ **Roll No:** ______________

Answer all questions in the respective spaces provided.
Use extra sheets for rough work. Any such extra sheet will not be corrected.

1. Which of the following assertions is/are true. Give one-line justifications. No credits will be given without proper reasoning. **(2×5)**

(a) The proposition $\neg p \Rightarrow q \vee r$ is equivalent to the proposition $\neg q \Rightarrow p \vee r$.

(b) The function $f : \mathbb{Q} \to \mathbb{N}$ that maps a/b with $\gcd(a, b) = 1$ to $a^2 + b^2$ is injective.

(c) Let $g : \mathbb{Z} \to \mathbb{Z}$ be a function satisfying $g(a + b) = g(a) + g(b)$ for all $a, b \in \mathbb{Z}$. Then $g(0) = 0$.

(d) Define a relation R on $\mathbb{N}$ as follows: Let $m, n \in \mathbb{N}$. Write $m = 2^s a$ and $n = 2^t b$ with a, b odd. Define $m \; R \; n$ if and only if $s \leqslant t$. Then R is a partial order on $\mathbb{N}$.

(e) If A and B are uncountable sets, then $A \cap B$ must be an uncountable set.

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2. Define a relation ρ on $\mathbb{R}$ as $a \; \rho \; b$ if and only if $a - b \in \mathbb{Q}$.

(a) Prove that ρ is an equivalence relation on $\mathbb{R}$. **(5)**

(b) Prove that the set $\mathbb{R}/\rho$ of equivalence classes of $\mathbb{R}$ under ρ is uncountable. **(5)**

(c) [*Take-home bonus*] Describe an explicit bijection between the sets $\mathbb{R}$ and $\mathbb{R}/\rho$. **(10)**

3. Use a diagonalization argument to prove that the set of all functions $\mathbb{N} \to \mathbb{N}$ is uncountable. No credit will be given to proofs that are not based on diagonalization arguments. **(5)**