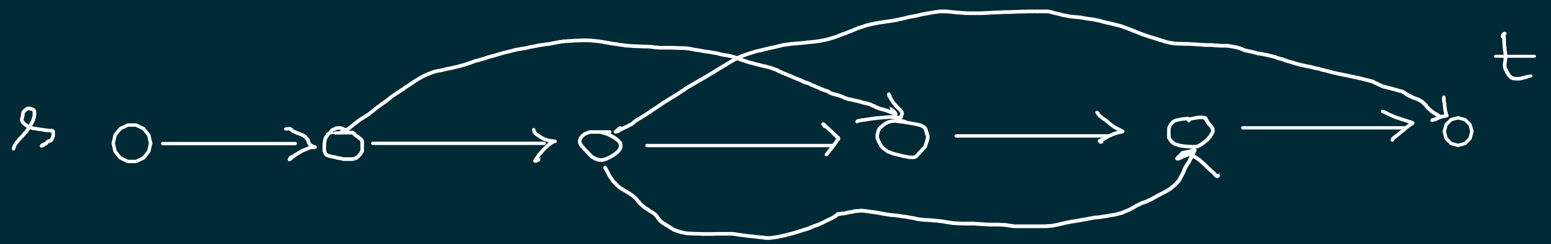
"Not all" instances of an NP-Complete problem are difficult.

DHAM-PATH easy for DAGs

DAGs directed acyclic graphs



topological sort

Longest and Shortest paths

Dijkstra, Bellman-Ford, Floyd-Warshall

They all fail if you allow -ve edge weights.

LONGEST_PATH $\leq$ SHORTEST_PATH
(+ve wts)

G, W, s, t $\quad\quad$ G, w', s, t

$$w'(u, v) = -w(u, v)$$

3CNFSAT is NP-complete

2CNFSAT $\in$ P

$$\phi = (a_1 \vee b_1) \wedge (a_2 \vee b_2) \wedge \dots \wedge (a_L \vee b_L)$$

$$G = (V, E)$$

$V \longrightarrow$ all variables and their complements

$a \vee b$ is a clause in ϕ

add the directed edges $\bar{a} \longrightarrow b$

and $\bar{b} \rightarrow a$

Theorem: ϕ is not satisfiable

$\iff$ G contains both an $x, \bar{x}$ path and an $\bar{x}, x$ path for some variable x.

[if] Any truth assignment

$x = 1$

$$\overset{1}{x} \to \quad \to \overset{1}{a} \to \overset{0}{b} \to \quad \to \overset{0}{\bar{x}}$$

$\bar{a} \vee b$ is a clause in ϕ

$0 \vee 0 = 0$

$x = 0$

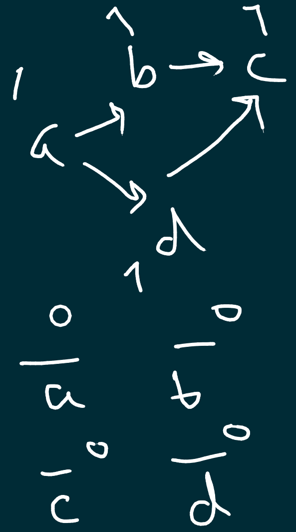
$$\overset{1}{\bar{x}} \qquad \underset{1}{a} \to \underset{0}{b} \qquad \overset{0}{x}$$

[only if]

G does not contain both $x,\bar{x}$ and $\bar{x},x$ paths for any variable x.

So long as all nodes in G are not assigned truth values, repeat:

Pick a literal a such that G does not contain an $a,\bar{a}$ path and a is not yet assigned a truth value.

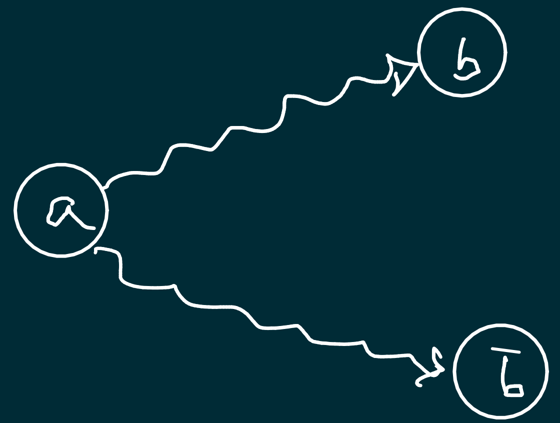


Assign truth value 1 to all nodes (including a itself) that are reachable from a, and the truth value 0 to the complements.

The truth assignment is well-defined.

$\exists$ literal b which we want to assign the truth value 0 and 1.

<u>Single iteration</u> (b is not assigned any value earlier)



a, b $\qquad$ $\bar{b}, \bar{a}$

$a, \bar{b}$ $\qquad$ $b, \bar{a}$

b was assigned earlier

$a, \bar{a}$ path

$1 \leftarrow$ now $\qquad$ $\bar{b}, \bar{a}$

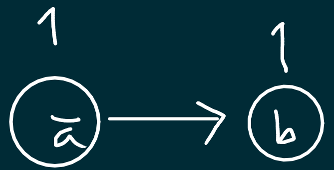
$0 \leftarrow$ earlier $\qquad$ $\bar{b} = 1$

This truth assignment satisfies ϕ

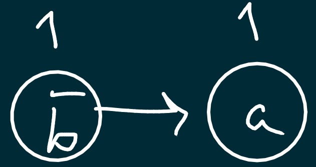
no such edges

Take $a \vee b$ in ϕ

$a = 0$



$b = 0$



DNFSAT $\in$ P

$\hookrightarrow$ sum of products

$$\phi = P_1 + P_2 + P_3 + \cdots + P_\ell$$

ϕ is satisfiable $\Leftrightarrow$ At least one P_i evaluates to 1

A product evaluates to 1

$\Leftrightarrow$ It does not contain both a variable x and its complement $\bar{x}$.

Eulerian-Tour problem

This is in P.

Hierholzer, 1873

G is Eulerian $\Leftrightarrow$ G is connected and every vertex of G has even degree.

Proof: See West.

How to compute an Eulerian tour.

Fleury's algorithm

Start at an arbitrary vertex v_1

Suppose $v_1, v_2, \ldots, v_i$ on a tour are generated.

Look at the remaining neighbors of v_i,

If only one, pick that as v_{i+1}.

If many nbrs, pick any such nbr as v_{i+1} provided that removing the edge (v_i, v_{i+1}) does not disconnect the graph.

Remove (v_i, v_{i+1})

If v_i has no more nbrs, delete v_i.

Not all NP-hard problems are NP-complete.

The Halting Problem is NP-hard.

Inputs
- A C program Q
- An input I for Q

Decide whether Q halts on I.

Diagonalization argument

HP is unsolvable

NP contains solvable problems only.

$HP \notin NP$.

HP is NP-Hard

$$\text{CNFSAT} \leq \text{HP}$$

$$\phi \longmapsto P, \phi$$

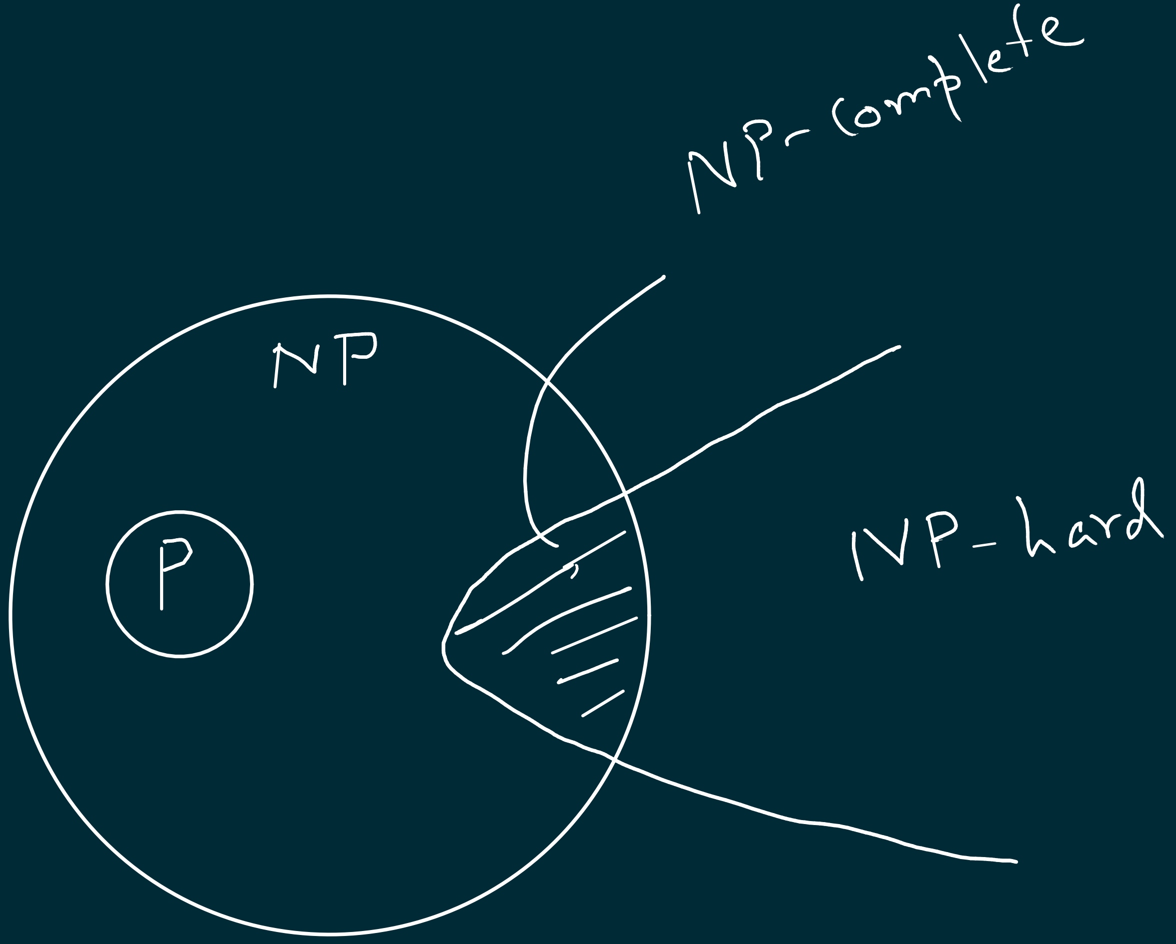
Count the no. n of variables in ϕ

write a fn to evaluate ϕ for any given truth assignment

write a loop $i = 0$ to $2^n - 1$

if ϕ evaluates to true on i, exit(0);

while (1);



If $P \neq NP$, all the inclusions are proper.