VERTEX - COVER

$G = (V, E)$ undirected graph

$U \subseteq V$ is called a vertex cover if for $(u, v) \in E$, either u or v (or both) belong to U.

minimum vertex cover

Decision version:

Given G and a positive integer ℓ decide whether G contains a vertex cover of size $|U| = \ell$ $(\leq \ell)$.

VERTEX-COVER $\in NP$

CNFSAT $\leq$ VERTEX-COVER

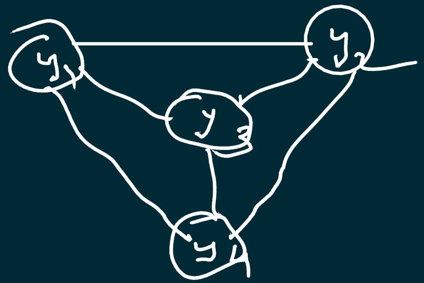
$$\phi \longmapsto (G, \ell)$$

variable gadget

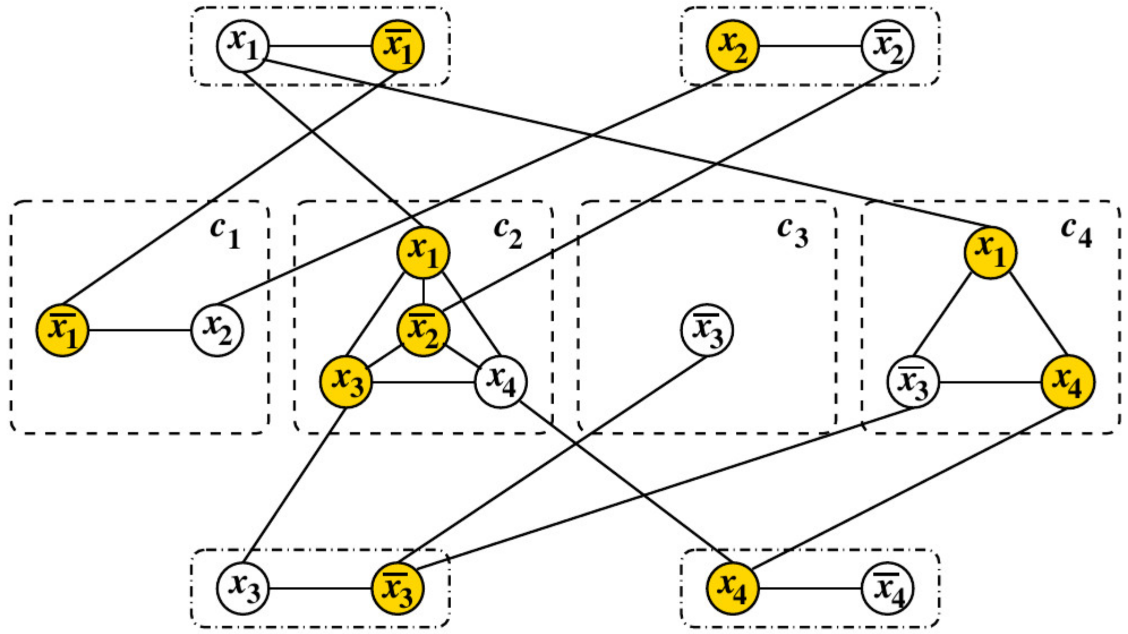
$y_1 \vee y_2 \vee \cdots \vee y_s$

clause gadget

complete graph on s vertices labeled by these literals



Converting $\phi = (\overline{x_1} \vee x_2) \wedge (x_1 \vee \overline{x_2} \vee x_3 \vee x_4) \wedge (\overline{x_3}) \wedge (x_1 \vee \overline{x_3} \vee x_4)$ to an undirected graph for proving the NP-Completeness of the vertex cover problem



n - # of variables (4)

m - # of clauses (4)

t - total # of literals in all the clauses (10)

ϕ is satisfiable

$x_1 = 0, x_2 = 1, x_3 = 0, x_4 = 1$ $\quad l = n + (t - m)$

CLIQUE

$G = (V, E)$ undirected

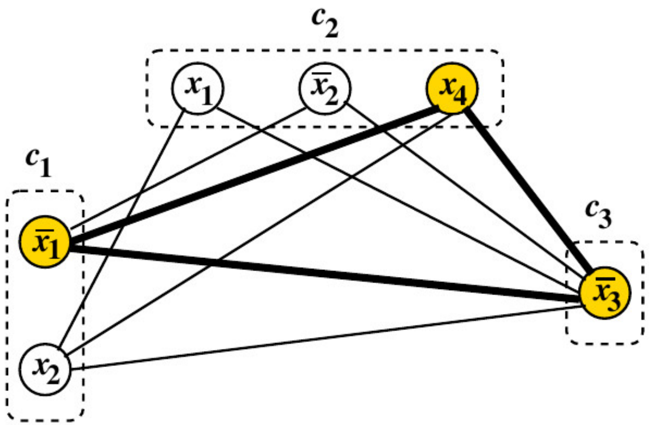
$U \subseteq V$ is called a clique in G
if $\forall u, v \in U$, $u \neq v$,
$(u, v) \in E$

Maximum clique problem

Decision version: Given G and ℓ,
determine whether G contains
a clique U of size $|U| = \ell$.

$\text{CNFSAT} \leq \text{CLIQUE}$

Converting $\phi = (\overline{x_1} \vee x_2) \wedge (x_1 \vee \overline{x_2} \vee x_4) \wedge (\overline{x_3})$ to an undirected graph for proving the NP-Completeness of the clique problem



l = # of clauses

ϕ is satisfiable — Each clause has a true literal

$x_1 = 0, \; x_2 = 1, \; x_3 = 0, \; x_4 = 1$

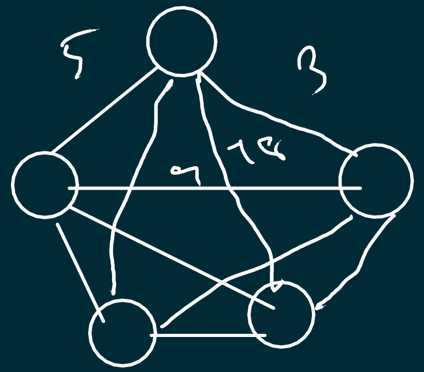
G has an l-clique — one and exactly one vertex from each clause gadget

$x_1 = 0, \; x_4 = 1, \; x_3 = 0$

$x_2 = 0$ or 1

TSP

Traveling Salesperson Problem



complete weighted graph on n vertices

To minimize the total cost of travel

G, c decide whether there is a tour of cost $\leq c$.

HAM-CYCLE $\leq$ TSP

$$G = (V, E) \longmapsto G' = (V', E'), \text{ weight}$$

$$V' = V$$

$e \in E'$, take $\quad wt(e) = \begin{cases} 1 & \text{if } e \in E \\ n+1 & \text{if } e \notin E \end{cases}$

$$c = n$$

LONGEST-PATH

G directed graph

s, t two vertices

edges have +ve weights.

Find the longest path from s to t in G.

G, s, t, ω, c, decide whether G contains an s, t path of total cost $\geq c$.

DHAM-PATH $\leq$ LONGEST-PATH

G, s, t G, s, t, w, c

$\omega(e) = 1 \ \forall e \in E$

$c = n - 1$

INDEPENDENT SET

$G = (V, E)$ undirected

$U \subseteq V$ is called independent

if $u, v \in U$ $(u \neq v)$ implies

$$(u, v) \notin E$$

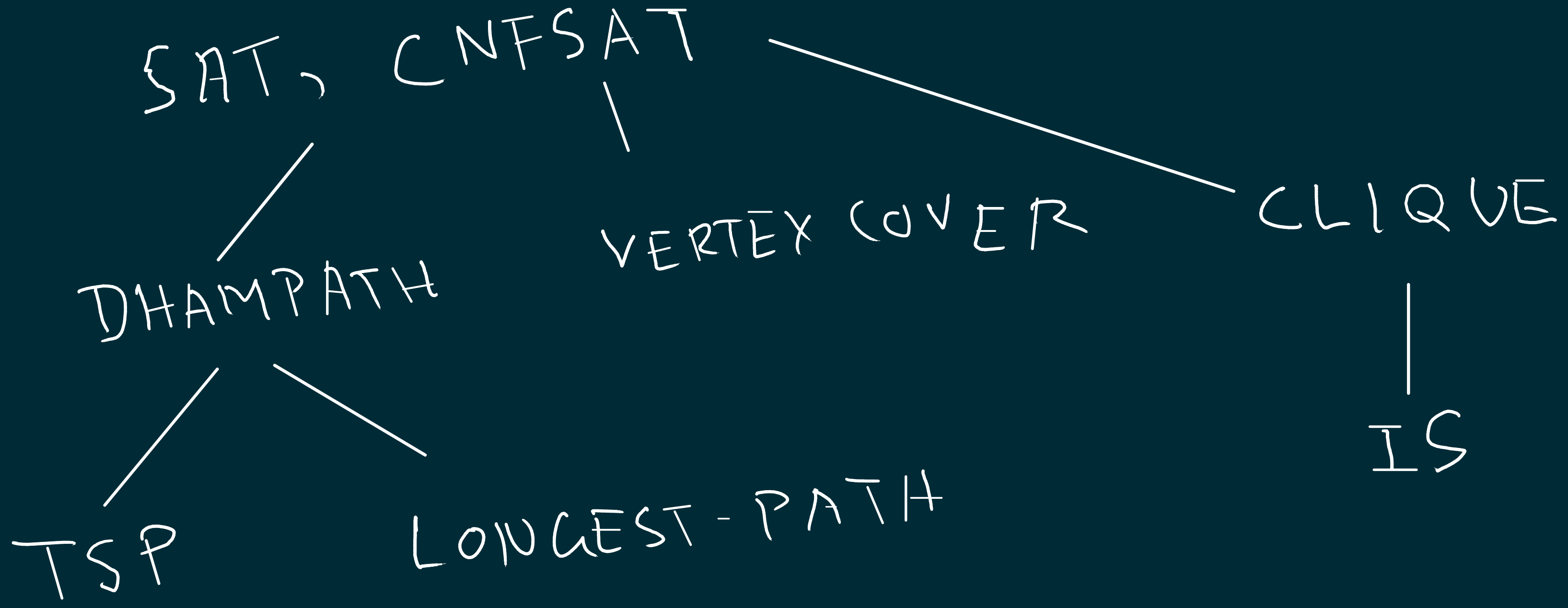
G, ℓ, decide whether G contains an IS U of size $|U| = \ell$.

CLIQUE $\leq$ IS

$(G, \ell) \mapsto (G', \ell)$

$G = (V, E)$

$G' = (V, \overline{E})$



<u>Big Question</u>: Knowing that Π is NPC, how can we "solve" Π ?