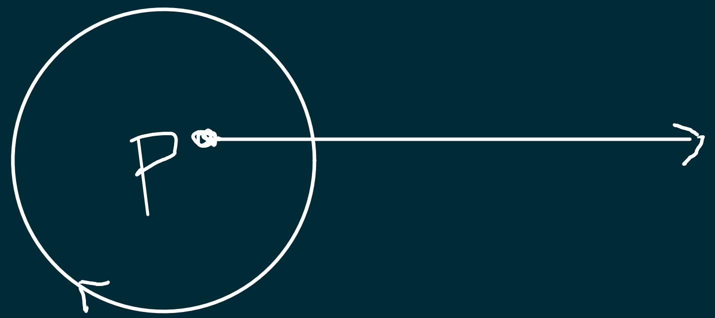
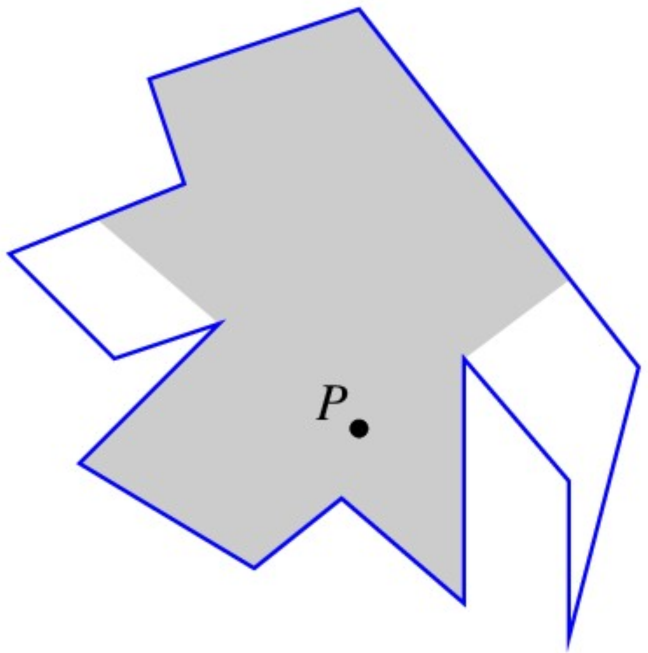
Visibility polygon

Ray sweep algorithm





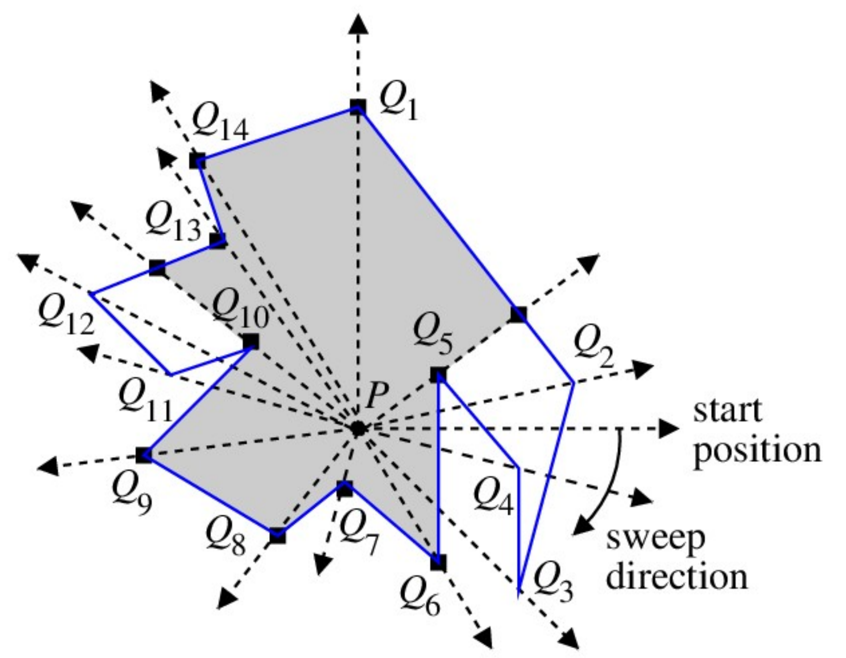
Region of visibility from P

Input: A simple polygon specified by a "clockwise" listing of the vertices.

A point P in the interior

Output: The region in the interior visible to P.

If the i/p is convex, the o/p is same as the i/p.



Event positions for the sweeping ray

Events: The corners of the i/p polygon.

S : sweep ray information

The set of active edges of the i/p polygon.

$Q = Q_i$

$Q_- = Q_{i-1}$

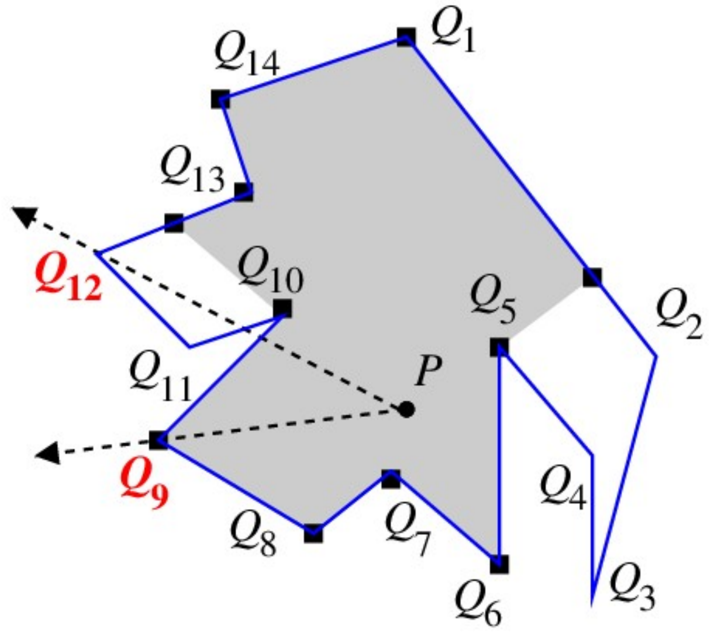
$Q_+ = Q_{i+1}$

$Q_-Q \to e_-$

$QQ_+ \to e_+$

Q_1 – LR Q_4 – RL

Q_3 – LL Q_5 – RR

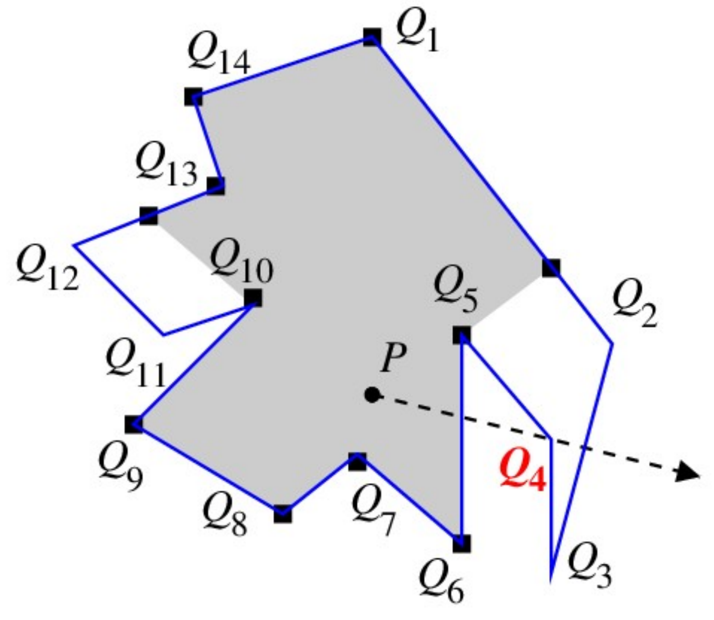


Events of type LR

S : Remove e_-
Insert e_+

Add Q to o/p if and only if e_+ is closest to P.

S is kept sorted with respect to their distances from P along the ray.

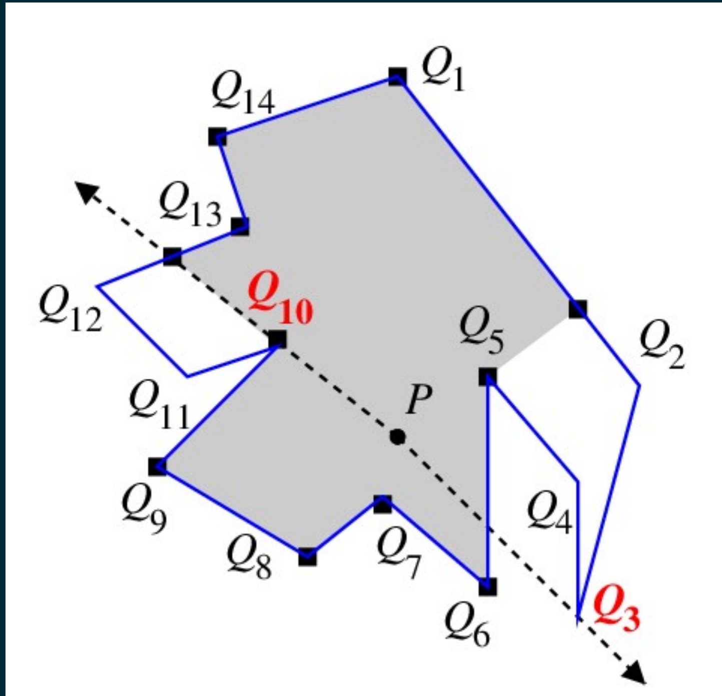


Events of Type RL

S: Insert e_-

Delete e_+

e_- cannot be closest to P.



Events of Type LL

Remove e_- and e_+ from S.

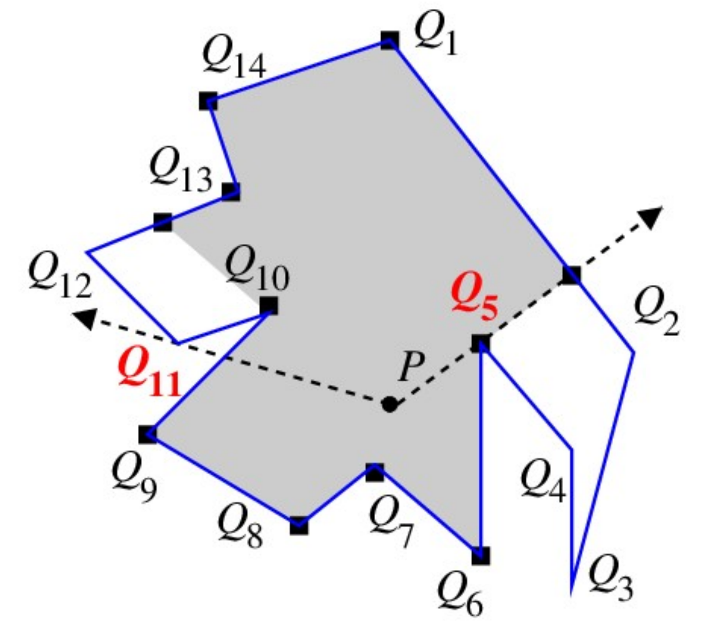
Before removal:

Check whether e_-/e_+ was closest to P

If not, done.

After removal: Find the intersection R of the ray with the closest active edge

o/p Q, R in that order



Events of Type RR

Insert e_- and e_+ to S.

Q_5 : e_+ then e_-

Q_{12} : e_- then e_+

After insertion : check whether e_-/e_+ is closest to P.

If yes, find the intersection R of the ray with the third nearest active line. O/p R and then Q

E – the vertices of the i/p polygon

findmin, deletemin

(order: angle w.r to the horizontal position)

priority queue (heap)

A sorted array is equally good.

S – insert, delete, findmin

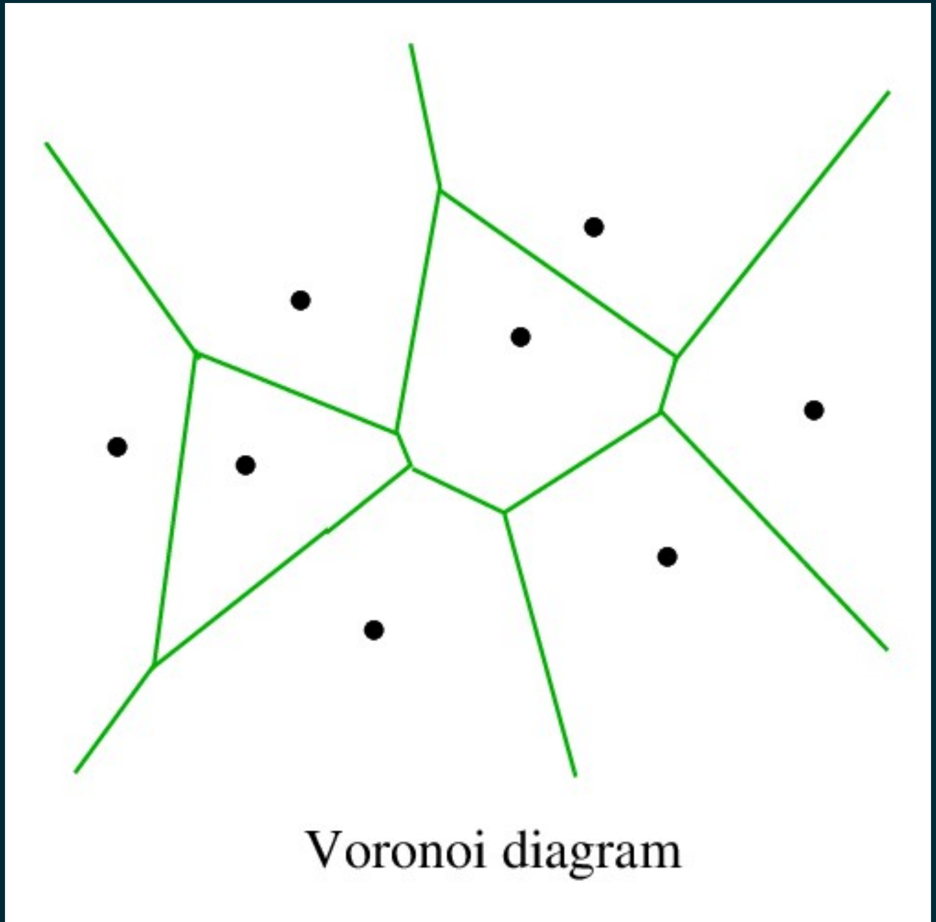
A height-balanced BST

Distances are not stored.

During ins/del, calculate the distances along only the ins/del path.

$O(n \log n)$

Voronoi Diagrams



Input: A finite set of points in gen'l pos S

- No three collinear
- No four on the same circle

$H_{ij} = \{P \mid d(P, P_i) \leq d(P, P_j)\}$

$\mathrm{Vcell}(P_i) = \bigcap_{j \neq i} H_{ij}$

$\mathrm{Vor}(S)$

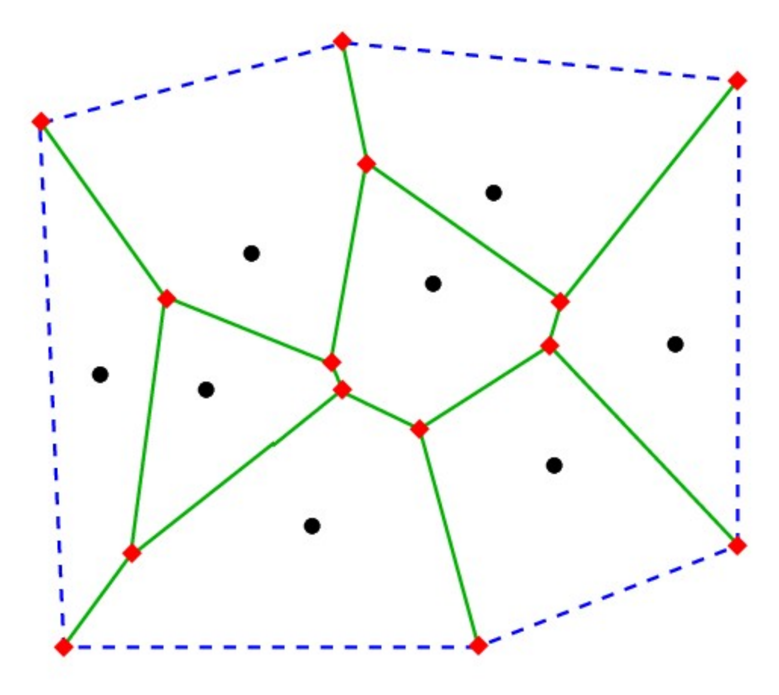
$O(n \log n)$ algo

There may be $\binom{n}{2}$ edges

in total

if every pair P_i, P_j contributes a portion of their perpendicular bisector

In reality, $Vor(S)$ consists only of $\Theta(n)$ edges.



Size of a Voronoi diagram

$G = (V, E)$

a planar graph

$n+1$ faces

Each vertex has degree 3.

$3V = 2E \qquad V = \frac{2}{3}E$

$F = n+1$

$V - E + F = 2$

$\frac{2}{3}E - E + n + 1 = 2$

$\frac{1}{3}E = n - 1$

$E = 3n - 3$

$V = 2n - 2$