CS31005: Algorithms-II

Class Test 1

September 28, 2020, 3-05 pm

Duration: 50 minutes for answering questions + 10 minutes for submission

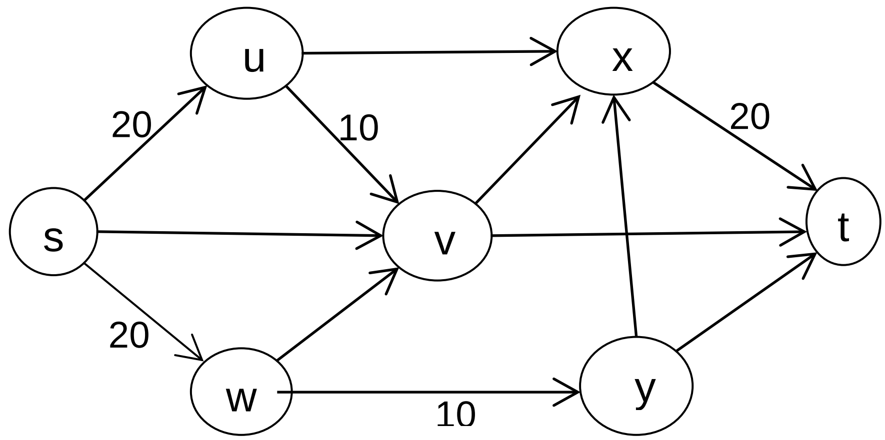
Answer all questions

All your answers MUST BE HANDWRITTEN on paper. Scan all papers with your answer in a SINGLE pdf and upload in the course page in Moodle. The size of the final pdf must be less than 10MB. You must upload the pdf strictly by 4-05 pm Moodle server time.

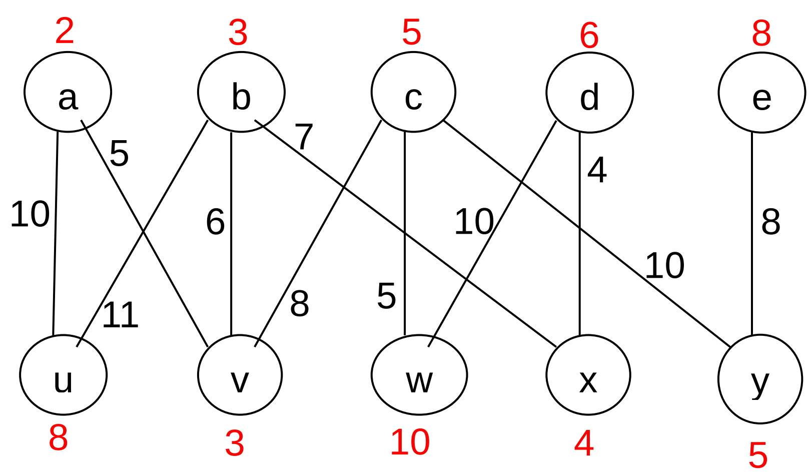
1. Do the following:
    a. Choose 12 distinct numbers x_1, x_2, …., x_{12} randomly from 1 and 100 (just pick without thinking). In your answer, show first the numbers x_1, x_2, …x_{12} clearly in one line, in the format x_1 = .., x_2 = .., and so on.
    b. Perform the following operations on an initially empty Fibonacci heap in this order: Insert x_5, Insert x_3, Insert x_7, Insert x_4, Insert x_8, Insert x_{11}, ExtractMin, Insert x_1, Insert x_9, Insert x_6, Insert x_{10}, Insert x_2, Insert x_{12}, ExtractMin. In any operation, whenever you have a choice between left and right for doing something, choose left. In your answer, (i) draw the heap after the first ExtractMin, and (ii) draw the heap after the second ExtractMin. Do not show anything else, and no explanations are needed. (10)

2. Consider the following flow network. The picture shows some of the capacities, you will need to fill up the other capacities choosing distinct integers randomly from 10 to 20. Starting with a flow of all 0's, show the first two steps of the Ford-Fulkerson algorithm with the following constraint: in the first step pick the longest augmenting path (largest number of edges), and in the second step, pick the augmenting path with the highest residual capacity.

    In your answer, clearly show (i) the original graph with all capacities filled up, (ii) the augmenting path chosen in the first step (write this as a sequence of vertices, do not mark on the graph), (iii) the flow network after the first augmentation, (iv) the corresponding residual graph after the first augmentation, (v) the augmenting path chosen in the second step, (vi) the flow network after the second augmentation, (vii) the corresponding residual graph after the second augmentation. Do not show anything else, and no explanations are needed. (10)



3. Consider the weighted bipartite graph below (dummy edges with weight 0 are not shown, but they are there). The picture shows the vertex labels (shown in red against each vertex) just after the completion of a relabeling step of the Hungarian algorithm. Let the matching at this stage be M = {(a,v), (b,x), (c,y)}. Show (i) the equality subgraph, (ii) the sets S, T, and the alternating tree found starting from the free vertex u until $N_\ell(S) = T$, (iii) the value of α_ℓ found (show the calculation), and (iv) the new labels of the vertices after the relabeling (just show the graph with the labels). (10)



4. A college has N students $X_1, X_2, \ldots, X_N$, M departments $D_1, D_2, \ldots, D_M$, and P societies $S_1, S_2, \ldots, S_P$. Each student is enrolled in exactly one department, and is a member of at least one society. The college has a student association (like your gymkhana) with one member from each society (You can assume that every society has at least one member). However, the society members have to be chosen such the student association has at most Q_k members from any department D_k. Design an algorithm to answer (yes or no) whether such a student association can be formed or not given the society memberships and the departments of the students. You must model the problem as a maximum flow problem.

Write clearly how you will construct the flow graph, draw the graph clearly identifying all vertices, edge capacities, and the source and the sink, and then say one sentence only on how you will answer yes or no from the maximum flow found. (15)

5. Consider the height function h in the preflow push method. Suppose that at some intermediate step, for some k, $0 < k < |V|$, there does not exist any vertex u with $h(u) = k$. Then prove that if there exists a vertex v with $h(v) > k$, then there cannot be any path from v to t in the residual graph. Write briefly, do not write an essay. (5)