Algorithms – II

Second Short Test

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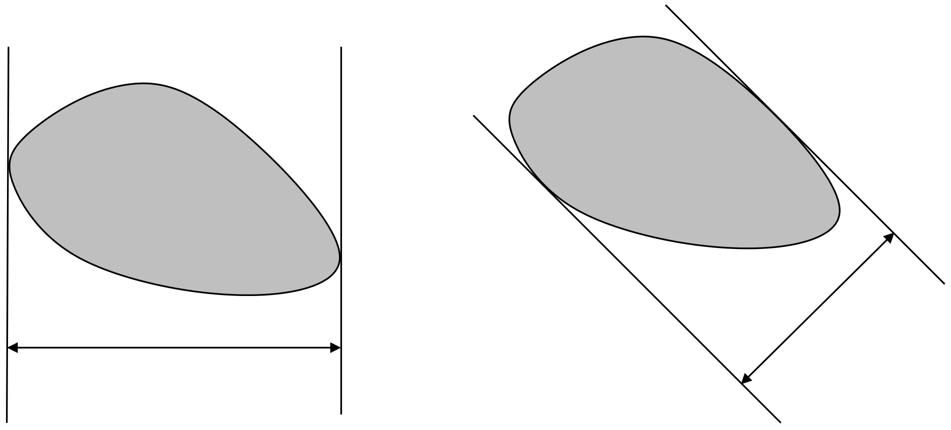
1. Which of the following regions in the two-dimensional plane is **not** convex?

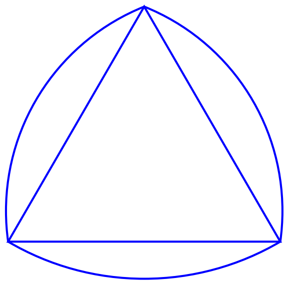
- **(A)** $\{(x,y) \mid y-5 \geqslant x+2\}$
- $\boxed{\textbf{(B)}}$ $\{(x,y) \mid |y-5| \geqslant |x+2|\}$
- **(C)** $\{(x,y) \mid y-5 \geqslant (x+2)^2\}$
- **(D)** $\{(x,y) \mid y-5 \geqslant \max[x+2,(x+2)^2]\}$

2. The width of a plane convex region along a direction is the distance between two parallel tangents to the region running perpendicular to the given direction. The following figure shows the horizontal width and the width at an angle of 45^0 with the horizontal of a convex region.



Let R be a plane convex region such that the width of R is the same in all directions. Which of the following is true about R?

- **(A)** R must be the region enclosed by a circle.
- **(B)** R must be the region enclosed by an ellipse.
- **(C)** R must be the convex hull of a finite set of points.
- $\boxed{\textbf{(D)}}$ R may be none of the regions in the other options.

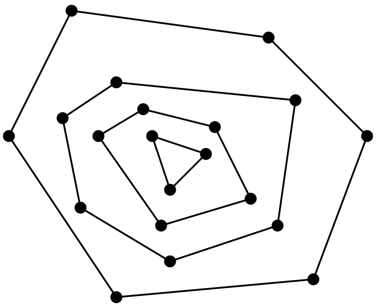


Reuleaux Triangle

3. Let S be a set of n points in the plane. You want to compute the rectangle of the smallest area and with sides parallel to the x- and y-axes, that encloses all the points in S. The points are not provided in any sorted order. What is the best complexity of solving this problem?

(A) $O(n \log n)$ $\boxed{\textbf{(B)}}$ $O(n)$ **(C)** $O(\log n)$ **(D)** $O(1)$

4. Let S be a set of n points in the plane. Let L_1 denote the set of vertices of the convex hull $\mathrm{CH}(S)$. We remove the points in L_1 from S, and compute the convex hull again. Now, let L_2 denote the set of vertices of $\mathrm{CH}(S \setminus L_1)$. Then, we delete the points of L_2 from $S \setminus L_1$, and repeat the same process until all points are removed. $L_1, L_2, L_3, \ldots$ are called the *onion layers* of S. The following figure describes this concept.



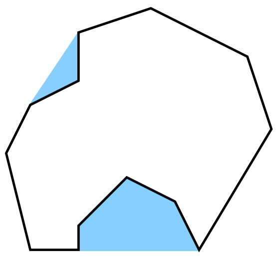
Suppose that we use Jarvis's march for each convex-hull construction. What is the worst-case running time for computing all the onion layers of S? Select the tightest bound.

(A) $\mathrm{O}(n^3)$ **(B)** $\mathrm{O}(n^{3/2})$ $\boxed{\textbf{(C)}}$ $\mathrm{O}(n^2)$ **(D)** $\mathrm{O}(n^2 \log n)$

5. Let S be a set of n points in the plane, and we compute the upper hull of S using Graham's scan. What is the maximum possible number of points that can be discarded from the current upper hull when a new point is considered in an iteration?

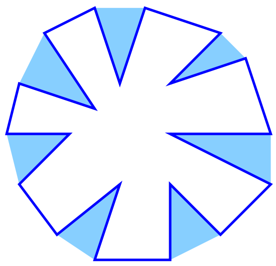
(A) $n-1$ $\boxed{\textbf{(B)}}$ $n-2$ **(C)** $n-3$ **(D)** $\lfloor n/2 \rfloor$

6. Let P be a simple polygon (not necessarily convex) of n vertices. Let H denote the convex hull of (the vertices) of P. Each connected region outside P but inside H is called a *pocket* of P. The following figure shows the two pockets that the polygon of the figure has.



What is the maximum number of pockets that P can have?

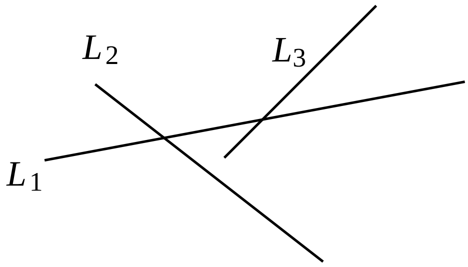
(A) $\Theta(1)$ **(B)** $\Theta(\log n)$ **(C)** $\Theta(\sqrt{n})$ $\boxed{\textbf{(D)}}$ $\Theta(n)$



The maximum can be

n / 2 (how?)

7. Suppose that we run the line-sweep algorithm for finding all the intersections of the following three line segments. The enter-segment, leave-segment and intersection events are denoted by $ES(L_i)$, $LS(L_i)$ and $\cap(L_i, L_j)$. In what order the events are processed by the algorithm (assuming that the sweep line is vertical and moves horizontally from $x = -\infty$ to $x = +\infty$)?



(A) $ES(L_1), ES(L_2), ES(L_3), \cap(L_1, L_2), \cap(L_1, L_3), LS(L_1), LS(L_2), LS(L_3)$
(B) $ES(L_1), ES(L_2), ES(L_3), \cap(L_1, L_2), \cap(L_1, L_3), LS(L_2), LS(L_3), LS(L_1)$
$\boxed{\textbf{(C)}}$ $ES(L_1), ES(L_2), \cap(L_1, L_2), ES(L_3), \cap(L_1, L_3), LS(L_2), LS(L_3), LS(L_1)$
(D) $ES(L_1), ES(L_2), \cap(L_1, L_2), ES(L_3), \cap(L_1, L_3), LS(L_1), LS(L_2), LS(L_3)$

8. Let S be a set of n sites **in general position**. The Voronoi cell of a site is called bounded if the cell has finite area. What is the maximum possible number of sites having bounded Voronoi cells?

(A) 3 **(B)** $\lfloor n/2 \rfloor$ $\boxed{\textbf{(C)}}$ $n-3$ **(D)** n

9. Let S be a set of n sites **not necessarily in general position**. The Voronoi cell of a site is called unbounded if the cell has infinite area. What is the maximum possible number of sites having unbounded Voronoi cells?

$\boxed{\textbf{(A)}}$ n **(B)** $n-3$ **(C)** $\lfloor n/2 \rfloor$ **(D)** 3

10. In Fortune's line-sweep algorithm for computing the Voronoi diagram of a set S of n sites, what is the minimum number of site events needed before the first circle event occurs?

Solution 3. (A circle event is defined by three (or more) sites. So before three site events, no circle event can occur.)