CS60045 Artificial Intelligence, Autumn 2023–2024

Class Test 2

06–November–2023 | 07:15pm–08:15pm | Maximum marks: 30

Roll no: ____________ **Name:** ____________________________

[*Write your answers in the question paper itself. Be brief and precise. Answer all questions.*]

1. [Predicate logic]

Encode the English statements of the four parts below, into well-formed predicate-logic formulas. Your encodings should use ***only*** the following predicates with the given meanings. **(2 × 4)**

boy(x) : x is a boy
girl(x) : x is a girl
love(x, y) : x loves y
marry(x, y) : x marries y
diff(x, y) : x and y are different

(a) Everyone loves everyone else except himself/herself.

Solution $\forall x \forall y \left[\mathrm{diff}(x,y) \Rightarrow \mathrm{love}(x,y) \right]$

(b) Every boy loves at least two girls.

Solution $\forall x \left[\mathrm{boy}(x) \Rightarrow \exists y \exists z \left[\mathrm{girl}(y) \wedge \mathrm{girl}(z) \wedge \mathrm{love}(x,y) \wedge \mathrm{love}(x,z) \wedge \mathrm{diff}(y,z) \right] \right]$

(c) Every boy who loves a girl does not love every other boy whom that girl loves.

Solution $\forall x \forall y \left[\left(\mathrm{boy}(x) \wedge \mathrm{girl}(y) \wedge \mathrm{love}(x,y) \right) \Rightarrow \forall z \left[\left(\mathrm{boy}(z) \wedge \mathrm{diff}(x,z) \wedge \mathrm{love}(y,z) \right) \Rightarrow \neg \mathrm{love}(x,z) \right] \right]$

(d) Every boy who loves a girl marries that girl, but not every girl who marries a boy loves that boy.

Solution $\left(\forall x \forall y \left[\left(\mathrm{boy}(x) \wedge \mathrm{girl}(y) \wedge \mathrm{love}(x,y) \right) \Rightarrow \mathrm{marry}(x,y) \right] \right) \wedge \left(\exists x \exists y \left[\mathrm{boy}(x) \wedge \mathrm{girl}(y) \wedge \mathrm{marry}(y,x) \wedge \neg \mathrm{love}(y,x) \right] \right)$

2. [Propositional logic]

There are four books on AI written respectively by Abra, Cadabra, Bingo, and Dingo. You have a limited budget, and can afford to buy only one book for your AI course. You want to buy a good one. You approach two seniors who give you the following suggestions.

Senior 1 Abra's book is not good. Moreover, if both Bingo's book and Cadabra's book are good, then Dingo's book is good too.

Senior 2 If Abra's book is good, then Cadabra's book is good too. However, it is not that all of the three books by Abra, Bingo, and Dingo are good.

Soon, you come to know from your friend that one of these two seniors is a liar, and the other is speaking the truth. Unfortunately, your friend cannot specify which of the two seniors is the liar.

(a) Your first task is to select a good book on AI, based upon the suggestions of the seniors and upon your friend's warning. You are required to solve this problem using a truth table (no other method is acceptable). Write the complete truth table, and justify your conclusion from the truth table. **(6)**

Solution We use the four proposition A, B, C, D to stand respectively for Abra's, Bingo's, Cadabra's, Dingo's book is good. We have the following responses of the two seniors.

$$S_1 = \left(\neg A\right) \bigwedge \left((B \wedge C) \Rightarrow D\right)$$
$$S_2 = \left(A \Rightarrow C\right) \bigwedge \left(\neg (A \wedge B \wedge D)\right)$$

Since exactly one of these two statements is true, the knowledge base is:

$$KB = S_1 \oplus S_2 = (S_1 \wedge \neg S_2) \vee (\neg S_1 \wedge S_2) = (S_1 \vee S_2) \wedge (\neg S_1 \vee \neg S_2)$$

The truth table is given below.

A	B	C	D	¬ A	(B ⋀ C) ⇒ D	S_1	A ⇒ C	¬ (A ⋀ B ⋀ D)	S_2	$S_1 \oplus S_2$
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	1	1	1	1	1	1	0
0	0	1	0	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	1	0
0	1	0	0	1	1	1	1	1	1	0
0	1	0	1	1	1	1	1	1	1	0
0	1	1	0	1	0	0	1	1	1	1
0	1	1	1	1	1	1	1	1	1	0
1	0	0	0	0	1	0	0	1	0	0
1	0	0	1	0	1	0	0	1	0	0
1	0	1	0	0	1	0	1	1	1	1
1	0	1	1	0	1	0	1	1	1	1
1	1	0	0	0	1	0	0	1	0	0
1	1	0	1	0	1	0	0	0	0	0
1	1	1	0	0	0	0	1	1	1	1
1	1	1	1	0	1	0	1	0	0	0

The knowledge base entails that **Cadabra's book is good** (each of the other three books may or may not be good).

Your second task is to prove the statement: "**Senior 1 is the liar**" (you do not have to show additionally that Senior 2 is not the liar). You must use the method of resolution-refutation (no other method is acceptable). Only in the rest of this exercise, you are allowed (but not forced) to use the switching symbols $+$, product (or juxtaposition), and $'$ to stand for $\vee$, $\wedge$, and $\neg$. For example, you may write $(x+yz')(u+v)'$ instead of $(x \vee (y \wedge \neg z)) \wedge \neg(u \vee v)$. Note that in the following two parts, you are **not** allowed to use the truth table or your conclusion of Part (a) in any manner.

(b) Derive all the clauses that the proof starts with. Show all your calculations. **(6)**

Solution We need to show that KB entails S_1'. Logical manipulations on KB give the following.

$$\begin{aligned}
S_1 &= A'(B'+C'+D) \\
S_2 &= (A'+C)(A'+B'+D') = A'+C(B'+D') = A'+B'C+CD' \\
S_1+S_2 &= A'((B'+C'+D)+1)+B'C+CD' = A'+B'C+CD' = (A'+C)(A'+B'+D') \\
S_1'+S_2' &= A+BCD'+A(C'+BD) = A(1+C'+BD)+BCD' = A+BCD' = (A+B)(A+C)(A+D')
\end{aligned}$$

Therefore the knowledge base gives us the following five clauses.

$$\begin{array}{ll}
[C1] & A'+C \\
[C2] & A'+B'+D' \\
[C3] & A+B \\
[C4] & A+C \\
[C5] & A+D'
\end{array}$$

Moreover, S_1 (the negation of the statement to prove) gives the following two clauses.

$$\begin{array}{ll}
[C6] & A' \\
[C7] & B'+C'+D
\end{array}$$

(c) Clearly show all the resolution steps to arrive at the null (that is, empty) clause. **(4)**

Solution The resolution steps that lead to contradiction are given below.

$$\begin{array}{rl}
[C8] & \mathrm{Res}_A(C_3,C_6) = B \\
[C9] & \mathrm{Res}_A(C_4,C_6) = C \\
[C10] & \mathrm{Res}_A(C_5,C_6) = D' \\
[C11] & \mathrm{Res}_B(C_7,C_8) = C'+D \\
[C12] & \mathrm{Res}_C(C_9,C_{11}) = D \\
[C13] & \mathrm{Res}_D(C_{10},C_{12}) = \mathbf{F}
\end{array}$$

3. [Planning]

There is a two-way infinite array of cells indexed by $c \in \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. A robot is initially stationed at $c = 0$. Exactly one cell at $c = t$ contains fabulous treasure. Assume that $t \neq 0$ (both $t > 0$ and $t < 0$ are allowed), and that t is known to the robot. At each step, the robot can move to an adjacent cell ($c - 1$ or $c + 1$ if it is at cell c). The task of the robot is to reach the t-th cell, collect the treasure there, and deposit the treasure in a locker at the cell $c = 0$. You need to pose this problem in the PDDL (planning domain definition language). For doing that, solve the following parts. Note that you are not asked to solve the problem by any planning algorithm. You are asked only to formulate the problem as a planning problem.

(a) What propositions do you use? **(2)**

Solution We use the following four propositions.

$$\begin{array}{rcl} \text{robotat}(c) & : & \text{The robot is at cell } c \\ \text{treasureat}(c) & : & \text{The treasure is at cell } c \\ \text{collected}() & : & \text{The treasure is collected.} \\ \text{deposited}() & : & \text{The treasure is deposited.} \end{array}$$

(b) What is/are the initial condition(s)? **(1)**

Solution $\text{robotat}(0) \wedge \text{treasureat}(t) \wedge \neg\text{collected}() \wedge \neg\text{deposited}()$

(c) What is/are the goal condition(s)? **(1)**

Solution $\text{deposited}()$

(d) What are the actions? (Mention only the preconditions, the add list, and the delete list for each action.) **(2)**

Solution We tabulate the actions along with the relevant details.

Action	**Preconditions**	**Delete list**	**Add list**
$\text{moveright}(c)$	$\text{robotat}(c)$	$\text{robotat}(c)$	$\text{robotat}(c+1)$
$\text{moveleft}(c)$	$\text{robotat}(c)$	$\text{robotat}(c)$	$\text{robotat}(c-1)$
$\text{collect}(c)$	$\text{robotat}(c) \wedge \text{treasureat}(c)$	$\text{treasureat}(c)$	$\text{collected}()$
$\text{deposit}(\,)$	$\text{robotat}(0) \wedge \text{collected}()$	—	$\text{deposited}()$