Let A be an array of n positive integers. A positive integer s is said to be a subset sum from A if there exist indices

 $0 \le i_1 \le i_2 \le \ldots \le i_k \le n-1$ 

such that

 $s = A[i_1] + A[i_2] + \ldots + A[i_k].$ 

Notice that the array A need not be sorted, and may contain duplicate entries. Moreover, a given sum s may be realized in multiple ways.

## Part I

Write a function to populate the array *A*. Pass the array *A* as the only parameter. The function first reads the number *n* of elements that the array will store  $(1 \le n \le 20)$ . The elements A[0], A[1], ..., A[n-1] are then supplied one by one by the user. The function should return the array size *n*.

## Part II

The user enters a positive integer s. You determine whether s is a subset sum from A. For this, write another function which takes three arguments: A, n and s. If s is a subset sum from A, the function prints a way in which the sum s is realized. If s is not a subset sum from A, a message is printed to that effect. This function does not return anything.

This function should implement the algorithm sketched here. Vary a counter *c* in the range  $[0, 2^{n-1}]$ . The *n*-bit binary representations of all these values of *c* are precisely all the bit strings of length *n*. Each bit string is naturally identified with a subset of the array indices. For example, let n = 5. The counter value  $c = 25 = 2^4 + 2^3 + 2^0 = (11001)_2$  is identified with the array indices  $\{0, 3, 4\}$ , and the subset sum corresponding to this is A[0] + A[3] + A[4]. If, for any of the  $2^n$  values of *c*, the subset sum equals *s*, then the search is successful.

In order to find the *n*-bit binary representation of *c*, you may repeatedly divide *n* by 2. Another possibility is to use bitwise operators. For example, the *i*-th bit of *c* is one if and only if  $c \& (1U \le i)$  is non-zero. Here  $\le$  is left shift by *i* bits, and & is the bit-wise AND operator.

## Part III

In this part, you write a <u>recursive</u> function in order to find out all subset sums coming from A. Let S be the sum of all the *n* elements of A. Since A consists of positive integers only, the subset sums from A must be in the range [0, S - 1] (here, the zero subset sum corresponds to the null set of indices). B is an array of size S + 1. The array entry B[s] is intended to store a count c in the range  $[0, 2^{n-1}]$  such that s is the subset sum corresponding to the count c (see Part II for the explanation). Each entry in B is initialized to -1. The recursive function takes the following arguments:

- *A* The input array
- *n* The size of A
- *i* An index in A
- *B* The array *B* as introduced above
- s A subset sum from the subarray (A[0], A[1], ..., A[i-1])
- *c* A count corresponding to the subset sum *s*

The function works as follows (in pseudocode). It modifies B[] as the only effect, and needs to return nothing.

If B[s] equals -1, record in B[s] the count c.

If no further recursion is possible, return.

Make the first recursive call in which A[i] is included in the subset sum.

Make the second recursive call in which A[i] is excluded from the subset sum.

After the recursive function returns to *main()* (or a wrapper function), look at the array *B*. For each index *s* in the range [0, S] with  $B[s] \neq -1$ , a subset realizing the sum *s* is stored as c = B[s]. Print the subset corresponding to *c*.

## Sample output

```
Enter array size (between 1 and 20): 5
A[0] = 6
A[1] = 4
A[2] = 7
A[3] = 4
A[4] = 9
Enter sum: 23
  23 = A[0] + A[1] + A[3] + A[4] = 6 + 4 + 4 + 9
21 different sums are possible:
    0
     4 = A[1] = 4
    6 = A[0] = 6
7 = A[2] = 7
    8 = A[1] + A[3] = 4 + 4
    9 = A[4] = 9
   10 = A[0] + A[1] = 6 + 4
   \begin{array}{l} 11 = A[1] + A[2] = 4 + 7 \\ 13 = A[0] + A[2] = 6 + 7 \end{array}
   14 = A[0] + A[1] + A[3] = 6 + 4 + 4
   15 = A[0] + A[4] = 6 + 9
16 = A[2] + A[4] = 7 + 9
   \begin{array}{c} 10 = A[2] + A[1] + A[2] = 6 + 4 + 7 \\ 19 = A[0] + A[1] + A[4] = 6 + 4 + 9 \\ 20 = A[1] + A[2] + A[4] = 4 + 7 + 9 \end{array}
   21 = A[0] + A[1] + A[2] + A[3] = 6 + 4 + 7 + 4
   22 = A[0] + A[2] + A[4] = 6 + 7 + 9
   22 = A[0] + A[2] + A[4] = 6 + 7 + 9

23 = A[0] + A[1] + A[3] + A[4] = 6 + 4 + 4 + 9

24 = A[1] + A[2] + A[3] + A[4] = 4 + 7 + 4 + 9

26 = A[0] + A[1] + A[2] + A[4] = 6 + 4 + 7 + 9

30 = A[0] + A[1] + A[2] + A[3] + A[4] = 6 + 4 + 7 + 4 + 9
```