CS13002 Programming and Data Structures, Spring 2004, Section 3/C

Laboratory Test II **Total points: 25** **March 19, 2004**

For students with odd PC numbers

Stirling numbers of the first kind

Stirling numbers $s(n,k)$ of the first kind are non-negative integers defined recursively as:

$$\begin{aligned}
s(0,0) &= 1, && (1)\\
s(n,0) &= 0 \quad \text{for } n > 0, && (2)\\
s(n,k) &= 0 \quad \text{for } k > n, && (3)\\
s(n,k) &= (n-1)s(n-1,k) + s(n-1,k-1) \quad \text{for } n > 0 \text{ and } 0 < k \leq n. && (4)
\end{aligned}$$

These numbers have many interesting combinatorial properties (See Graham, Knuth and Patashnik's book *Concrete Mathematics*). In this exercise you are asked to compute the value of $s(n,k)$ in two different ways.

Method 1

Write a recursive function to compute $s(n,k)$. Note that Equation (4) defines the basic recursion, whereas the boundary conditions are provided by Equations (1)–(3).

Method 2

Write an iterative function to compute $s(n,k)$. You should better maintain a two-dimensional array and compute the values $s(n,k)$ in a particular order of the pair (n,k).

Output

Show the values of $s(n,k)$ for $0 \leq n \leq 9$ and $0 \leq k \leq n$, computed using both the recursive and the iterative methods. Your output should be presented in the following neat format. Note that each line in your print-out can hold eighty characters. Manage the number of spaces accordingly.

```
Recursive method
 n |  s(n,0) s(n,1) s(n,2) s(n,3) s(n,4) s(n,5) s(n,6) s(n,7) s(n,8) s(n,9)
---+------------------------------------------------------------------------
 0 |      x
 1 |      x      x
 2 |      x      x      x
 3 |      x      x      x      x
 4 |      x      x     xx      x      x
 5 |      x     xx     xx     xx     xx      x
 6 |      x    xxx    xxx    xxx     xx     xx      x
 7 |      x    xxx   xxxx   xxxx    xxx    xxx     xx      x
 8 |      x   xxxx  xxxxx  xxxxx   xxxx   xxxx    xxx     xx      x
 9 |      x  xxxxx xxxxxx xxxxxx  xxxxx  xxxxx   xxxx    xxx     xx      x
----------------------------------------------------------------------------

Iterative method
 n |  s(n,0) s(n,1) s(n,2) s(n,3) s(n,4) s(n,5) s(n,6) s(n,7) s(n,8) s(n,9)
---+------------------------------------------------------------------------
 0 |      x
 1 |      x      x
 2 |      x      x      x
 3 |      x      x      x      x
 4 |      x      x     xx      x      x
 5 |      x     xx     xx     xx     xx      x
 6 |      x    xxx    xxx    xxx     xx     xx      x
 7 |      x    xxx   xxxx   xxxx    xxx    xxx     xx      x
 8 |      x   xxxx  xxxxx  xxxxx   xxxx   xxxx    xxx     xx      x
 9 |      x  xxxxx xxxxxx xxxxxx  xxxxx  xxxxx   xxxx    xxx     xx      x
----------------------------------------------------------------------------
```

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Laboratory Test II **Total points: 25** **March 19, 2004**

For students with <u>even</u> PC numbers

Stirling numbers of the second kind

Stirling numbers $S(n,k)$ of the second kind are non-negative integers defined recursively as:

$$
\begin{aligned}
S(0,0) &= 1, & (1)\\
S(n,0) &= 0 \quad \text{for } n > 0, & (2)\\
S(n,k) &= 0 \quad \text{for } k > n, & (3)\\
S(n,k) &= kS(n-1,k) + S(n-1,k-1) \quad \text{for } n > 0 \text{ and } 0 < k \le n. & (4)
\end{aligned}
$$

These numbers have many interesting combinatorial properties (See Graham, Knuth and Patashnik's book *Concrete Mathematics*). In this exercise you are asked to compute the value of $S(n,k)$ in two different ways.

Method 1

Write a recursive function to compute $S(n,k)$. Note that Equation (4) defines the basic recursion, whereas the boundary conditions are provided by Equations (1)–(3).

Method 2

Write an iterative function to compute $S(n,k)$. You should better maintain a two-dimensional array and compute the values $S(n,k)$ in a particular order of the pair (n,k).

Output

Show the values of $S(n,k)$ for $0 \le n \le 9$ and $0 \le k \le n$, computed using both the recursive and the iterative methods. Your output should be presented in the following neat format. Note that each line in your print-out can hold eighty characters. Manage the number of spaces accordingly.

```
Recursive method
 n |  S(n,0) S(n,1) S(n,2) S(n,3) S(n,4) S(n,5) S(n,6) S(n,7) S(n,8) S(n,9)
---+-----------------------------------------------------------------------
 0 |      x
 1 |      x      x
 2 |      x      x      x
 3 |      x      x      x      x
 4 |      x      x      x      x      x
 5 |      x      x     xx     xx     xx      x
 6 |      x      x     xx     xx     xx     xx      x
 7 |      x      x     xx    xxx    xxx    xxx     xx      x
 8 |      x      x    xxx    xxx   xxxx   xxxx    xxx     xx      x
 9 |      x      x    xxx   xxxx   xxxx   xxxx   xxxx    xxx     xx      x
---------------------------------------------------------------------------
```

```
Iterative method
 n |  S(n,0) S(n,1) S(n,2) S(n,3) S(n,4) S(n,5) S(n,6) S(n,7) S(n,8) S(n,9)
---+-----------------------------------------------------------------------
 0 |      x
 1 |      x      x
 2 |      x      x      x
 3 |      x      x      x      x
 4 |      x      x      x      x      x
 5 |      x      x     xx     xx     xx      x
 6 |      x      x     xx     xx     xx     xx      x
 7 |      x      x     xx    xxx    xxx    xxx     xx      x
 8 |      x      x    xxx    xxx   xxxx   xxxx    xxx     xx      x
 9 |      x      x    xxx   xxxx   xxxx   xxxx   xxxx    xxx     xx      x
---------------------------------------------------------------------------
```