# CS13002 Programming and Data Structures, Spring 2004, Section 3/C

**Laboratory Test I** **Total points: 20** **February 13, 2004**

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**For students with <u>odd</u> PC numbers**

**<u>Calculation of $\pi$</u>**

We know that the power series for $\frac{1}{1+x^2}$ is

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots + (-1)^n x^{2n} + \cdots \qquad \text{valid for } |x| < 1. \tag{1}$$

Integration of both sides gives

$$\tan^{-1} x = \int_0^x \frac{\mathrm{d}x}{1+x^2} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots , \tag{2}$$

a formula that holds for $|x| \leq 1$. Putting $x = 1$ yields

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{(-1)^n}{2n+1} + \cdots . \tag{3}$$

Truncating the series after the $n$-th term gives an approximate value for $\pi$. Better approximations result for larger values of $n$. However, the series on the RHS of (3) converges pretty slowly. A series that converges faster is obtained by putting $x = \frac{1}{\sqrt{3}}$ in (2):

$$\frac{\pi}{6} = \frac{1}{\sqrt{3}} \left[ 1 - \frac{1}{3 \times 3} + \frac{1}{3^2 \times 5} - \frac{1}{3^3 \times 7} + \cdots + \frac{(-1)^n}{3^n \times (2n+1)} + \cdots \right]. \tag{4}$$

Write a program to compute the approximate values of $\pi$ from the partial sums of zeroth to $n$-th terms of (3) and (4), where $n = 10^i$, $i = 1, 2, \ldots, 6$. Your output should be presented in the following format:

```
n =      10, approximation1 = x.xxxxxxxxxx, approximation2 = x.xxxxxxxxxx
n =     100, approximation1 = x.xxxxxxxxxx, approximation2 = x.xxxxxxxxxx
n =    1000, approximation1 = x.xxxxxxxxxx, approximation2 = x.xxxxxxxxxx
n =   10000, approximation1 = x.xxxxxxxxxx, approximation2 = x.xxxxxxxxxx
n =  100000, approximation1 = x.xxxxxxxxxx, approximation2 = x.xxxxxxxxxx
n = 1000000, approximation1 = x.xxxxxxxxxx, approximation2 = x.xxxxxxxxxx
```

**Note:** (1) The correct value of $\pi$ rounded to 10 places of decimal is $3.1415926536$ and can be obtained by the math library call `4 * atan(1)`.

(2) You are recommended to use `double` variables instead of `float` variables.

(3) You are recommended to print floating point values with a precision of ten decimal digits.

(4) Do **<u>not</u>** print the output for any value of $n$ other than the six values mentioned above.

(5) You must **<u>not</u>** use any math library call (like `pow` or `sqrt`). Use the value $1.7320508076$ for $\sqrt{3}$. You may maintain a temporary variable whose value is updated in the loop using the formula $3^n = 3^{n-1} \times 3$ or $(-3)^n = (-3)^{n-1} \times (-3)$.

(6) Do **<u>not</u>** use arrays.

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**For students with <u>even</u> PC numbers**

## Calculation of $\pi$

We know that the power series for $\frac{1}{\sqrt{1-x^2}}$ is

$$\frac{1}{\sqrt{1-x^2}} = 1 + \left(\frac{1}{2 \times 1!}\right) x^2 + \left(\frac{1 \times 3}{2^2 \times 2!}\right) x^4 + \cdots + \left(\frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{2^n \times n!}\right) x^{2n} + \cdots$$

and is valid of $|x| < 1$. Integration of both sides gives

$$\begin{aligned} \sin^{-1} x &= \int_0^x \frac{\mathrm{d}x}{\sqrt{1-x^2}} \\ &= x + \left(\frac{1}{2 \times 1!}\right) \frac{x^3}{3} + \left(\frac{1 \times 3}{2^2 \times 2!}\right) \frac{x^5}{5} + \cdots + \left(\frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{2^n \times n!}\right) \frac{x^{2n+1}}{2n+1} + \cdots . \end{aligned}$$

Putting $x = \frac{1}{2}$ yields

$$\frac{\pi}{6} = \frac{1}{2} + \frac{1}{3}\left(\frac{1}{2^4 \times 1!}\right) + \frac{1}{5}\left(\frac{1 \times 3}{2^7 \times 2!}\right) + \cdots + \frac{1}{2n+1}\left(\frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{2^{3n+1} \times n!}\right) + \cdots .$$

Truncating the series after the $n$-th term gives an approximate value for $\pi$. Better approximations result for larger values of $n$. Let $s_n$ be the partial sum of the series till the $n$-th term (starting from the zeroth term $\frac{1}{2}$).

Write a program that computes successively better approximations of $\pi$ generated from $s_0, s_1, s_2, \ldots$. Stop when two consecutive approximations differ by less than $1/10^{11}$. Your output should be presented in the following format:

```
n = 0, s_n = x.xxxxxxxxxx, approximate value of pi = x.xxxxxxxxxx
n = 1, s_n = x.xxxxxxxxxx, approximate value of pi = x.xxxxxxxxxx
n = 2, s_n = x.xxxxxxxxxx, approximate value of pi = x.xxxxxxxxxx
...
```

**Note:** (1) The correct value of $\pi$ rounded to 10 places of decimal is 3.1415926536 and can be obtained by the math library call `6 * asin(0.5)`.

(2) You are recommended to use `double` variables instead of `float` variables.

(3) You are recommended to print floating point values with a precision of ten decimal digits.

(4) You must **<u>not</u>** use any math library call (like `pow`). You may maintain in a temporary variable the value $t_n = \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{2^{3n+1} \times n!}$ computed as $t_n = \frac{t_{n-1} \times (2n-1)}{8n}$ from the previous value $t_{n-1}$.

(5) Do **<u>not</u>** use arrays.