# CS69001 Computing Laboratory – I

## Practice Exercises: Set 3

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**1.** Let $G = (V, E)$ be a graph. Declare a data type to store $G$ in the adjacency-matrix representation. The user supplies $n = |V|$ and $m = |E|$, and then $m$ pairs $(u, v)$. Assume that the vertices are numbered $0, 1, 2, \ldots, n-1$. Construct $G$ in the following two cases.

**(a)** $G$ is a directed graph.

**(b)** $G$ is an undirected graph.

**2.** Repeat the last exercise for the adjacency-list representation.

**3.** Write a function that, given a graph $G = (V, E)$, finds and prints all the paths in $G$ of length three. Your function should run in $O(|V|^4)$ time. Write the function for both the adjacency-matrix and the adjacency-list representations of $G$.

**4.** Let $G = (V, E)$ be a directed graph with $|V| = n$. A node $v \in V$ is called a universal sink in $G$ if $(u, v) \in E$ for all $u \in V$, $u \neq v$, and $(v, w) \notin E$ for all $w \in V$, $w \neq v$. Write an $O(n)$-time function to identify the universal sink of $G$ if it exists or report that no such node exists. Which representation of $G$ would you prefer?

**5.** **(a)** Write a function to return a BFS tree $T$ of an undirected graph $G = (V, E)$. Store $T$ in an array in the parent representation.

**(b)** Write a function that, given $v \in V$, finds and prints the unique path in $T$ from the root of $T$ to $v$.

**(c)** Write a function that, given two nodes $u, v \in V$, finds and prints the unique path from $u$ to $v$ in $T$.

**6.** The user supplies a positive integer $N \geqslant 10$. Construct a directed acyclic graph (DAG) $G = (V, E)$ with $V = \{1, 2, 3, \ldots, N\}$, and with $(a, b) \in E$ if and only if $a$ is a proper divisor of $b$. Write a function to produce a topological sorting of $V$.

**7.** Consider the DAG $G = (V, E)$ of the last exercise. Write an efficient function that computes the number of directed paths from $a$ to $b$ for every $a, b \in V$.