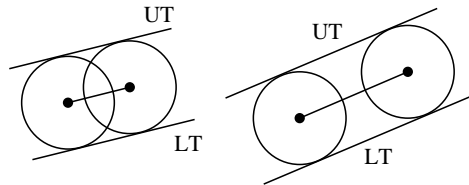


CS69001 Computing Laboratory – I

Practice Exercises: Set 1

1. Propose a user-defined data type to represent a point in the two-dimensional plane. Write a function that, given three points A, B, C as input, returns the area of the triangle ABC .
2. Let C_1 and C_2 be two circles of the same radius r with centers at (c_1, d_1) and (c_2, d_2) , respectively. Assume that $c_1 \neq c_2$ and $d_1 \neq d_2$. Write a program to compute the common upper and lower tangents to the circles.



3. Write an $O(n)$ -time function that, given a positive integer n as input, finds and prints a narrowest interval $[i, j]$ such that $\sum_{r=i}^j \binom{n}{r} \geq 0.9 \times 2^n$. (**Hint:** Search around the center.)

4. Catalan numbers $C(n)$, $n \geq 0$, are defined recursively as follows.

$$C(0) = 1,$$

$$C(n) = C(0)C(n-1) + C(1)C(n-2) + C(2)C(n-2) + \cdots + C(n-1)C(0) \text{ for } n \geq 2.$$

- (a) Write a recursive function to return $C(n)$.
- (b) Write a non-recursive function to return $C(n)$.

Note: Use the given formulas for Catalan numbers. Do not use other formulas like $C(n) = \binom{2n}{n} / (n+1)$.

5. Write a function (may be recursive) to print the Gray code of length n . For example, for $n = 3$, you should print the following sequence of codewords: 000, 001, 011, 010, 110, 111, 101, 100.
6. Let $n \geq k \geq 0$ be integers. Write a recursive function to print all the k -element subsets of $\{1, 2, 3, \dots, n\}$.
7. You are given an $n \times n$ matrix M and a $k \times k$ matrix P of bits for some $n \geq k \geq 0$. Write a function to find a $k \times k$ submatrix (contiguous) of M , which has the smallest Hamming distance with P .
8. [Eight-Queens puzzle] You are given an 8×8 chessboard. Your task is to place eight queens on the board such that no queen can attack any other queen, that is, no two of the queens share the same row, column or diagonal (forward or backward). Generalize the problem to an $n \times n$ board.
9. Large integers like 31415926535897932384626433832795028841971693993751 can be represented by a linked list of its digits (with the least significant digit at the beginning of the list).
 - (a) Write a function that reads a positive integer as a character string of decimal digits, and returns a linked list storing the number in the format mentioned above.
 - (b) Write a function to add two large integers in the above format.
 - (c) Write a function to multiply two large integers in the above format.
10. In a *circular linked list*, the next pointer of the last node points back to the first node in the list. Write a function that, given a circular linked list and a positive integer k , replaces the key at each node by the maximum of the next k elements in the list. For example, if the input list is 3, 1, 6, 2, 7, 4, 5, 2, 9, 8, 5, 2, and $k = 3$, the list should be updated to 6, 7, 7, 7, 5, 9, 9, 9, 8, 5, 3, 6. Notice that no new linked list is to be created. The existing list must be modified.