Graph Applications

You have three glasses of capacities c_1, c_2, c_3 ml. Initially, the three glasses contain a_1, a_2, a_3 ml of l'eau. Eventually, you want the glasses to contain b_1, b_2, b_3 ml of l'eau. Assume that the capacities c_1, c_2, c_3 are positive integers, whereas a_i, b_i are non-negative integers. We must have $0 \leqslant a_i \leqslant c_i$ and $0 \leqslant b_i \leqslant c_i$ for all $i = 1, 2, 3$. Assume further that $a_1 + a_2 + a_3 = b_1 + b_2 + b_3 = T$ (say).

You have no measuring devices nor any source or sink of l'eau. You can pour l'eau from a non-empty glass to another glass (not full). You must stop until either the first glass becomes empty or the second glass becomes full (whichever happens earlier). Let u_1, u_2, u_3 (in ml) be the amounts of l'eau in the three glasses after a move. Then, you must have $u_i = 0$ or $u_i = c_i$ for at least one $i \in \{1, 2, 3\}$. Moreover, we must have $u_1 + u_2 + u_3 = T$. Assume that the final contents b_1, b_2, b_3 are of this particular type (otherwise you do not have a solution in general). The initial contents a_1, a_2, a_3, however, may or may not be of this form (because these initial amounts are supplied to you by the quiz master, although you must have $a_1 + a_2 + a_3 = T$).

Your task is to find out whether by making moves of the above form (pouring l'eau from one glass to another), you can convert the contents of the three glasses from a_1, a_2, a_3 to b_1, b_2, b_3. If it is possible, you should also find out a shortest sequence of moves, that realizes this change of contents.

Pose this puzzle as a graph-theoretic problem, and solve the puzzle by an efficient algorithm. The details are to be designed by you. You may use a transcript similar to that shown in the sample output.

Sample output

```
Glass capacities : 27 28 20
Initial contents : 15 13  7
Final contents   :  5 10 20

+++ The vertices are
    ( 0,15,20) ( 0,16,19) ( 0,17,18) ( 0,18,17) ( 0,19,16) ( 0,20,15) ( 0,21,14)
    ( 0,22,13) ( 0,23,12) ( 0,24,11) ( 0,25,10) ( 0,26, 9) ( 0,27, 8) ( 0,28, 7)
    ( 1,14,20) ( 1,28, 6) ( 2,13,20) ( 2,28, 5) ( 3,12,20) ( 3,28, 4) ( 4,11,20)
    ( 4,28, 3) ( 5,10,20) ( 5,28, 2) ( 6, 9,20) ( 6,28, 1) ( 7, 8,20) ( 7,28, 0)
    ( 8, 7,20) ( 8,27, 0) ( 9, 6,20) ( 9,26, 0) (10, 5,20) (10,25, 0) (11, 4,20)
    (11,24, 0) (12, 3,20) (12,23, 0) (13, 2,20) (13,22, 0) (14, 1,20) (14,21, 0)
    (15, 0,20) (15,13, 7) (15,20, 0) (16, 0,19) (16,19, 0) (17, 0,18) (17,18, 0)
    (18, 0,17) (18,17, 0) (19, 0,16) (19,16, 0) (20, 0,15) (20,15, 0) (21, 0,14)
    (21,14, 0) (22, 0,13) (22,13, 0) (23, 0,12) (23,12, 0) (24, 0,11) (24,11, 0)
    (25, 0,10) (25,10, 0) (26, 0, 9) (26, 9, 0) (27, 0, 8) (27, 1, 7) (27, 2, 6)
    (27, 3, 5) (27, 4, 4) (27, 5, 3) (27, 6, 2) (27, 7, 1) (27, 8, 0)

+++ Graph generated
    ( 0,15,20) -> ( 0,28, 7) (15, 0,20) (20,15, 0)
    ( 0,16,19) -> ( 0,15,20) ( 0,28, 7) (16, 0,19) (19,16, 0)
    ( 0,17,18) -> ( 0,15,20) ( 0,28, 7) (17, 0,18) (18,17, 0)
    ( 0,18,17) -> ( 0,15,20) ( 0,28, 7) (17,18, 0) (18, 0,17)
    ( 0,19,16) -> ( 0,15,20) ( 0,28, 7) (16,19, 0) (19, 0,16)
    ( 0,20,15) -> ( 0,15,20) ( 0,28, 7) (15,20, 0) (20, 0,15)
    ( 0,21,14) -> ( 0,15,20) ( 0,28, 7) (14,21, 0) (21, 0,14)
    ( 0,22,13) -> ( 0,15,20) ( 0,28, 7) (13,22, 0) (22, 0,13)
    ( 0,23,12) -> ( 0,15,20) ( 0,28, 7) (12,23, 0) (23, 0,12)
    ( 0,24,11) -> ( 0,15,20) ( 0,28, 7) (11,24, 0) (24, 0,11)
    ( 0,25,10) -> ( 0,15,20) ( 0,28, 7) (10,25, 0) (25, 0,10)
    ( 0,26, 9) -> ( 0,15,20) ( 0,28, 7) ( 9,26, 0) (26, 0, 9)
    ( 0,27, 8) -> ( 0,15,20) ( 0,28, 7) ( 8,27, 0) (27, 0, 8)
    ( 0,28, 7) -> ( 0,15,20) ( 7,28, 0) (27, 1, 7)
    ( 1,14,20) -> ( 0,15,20) ( 1,28, 6) (15, 0,20) (21,14, 0)
    ( 1,28, 6) -> ( 0,28, 7) ( 1,14,20) ( 7,28, 0) (27, 2, 6)
    ( 2,13,20) -> ( 0,15,20) ( 2,28, 5) (15, 0,20) (22,13, 0)
    ( 2,28, 5) -> ( 0,28, 7) ( 2,13,20) ( 7,28, 0) (27, 3, 5)
    ( 3,12,20) -> ( 0,15,20) ( 3,28, 4) (15, 0,20) (23,12, 0)
    ( 3,28, 4) -> ( 0,28, 7) ( 3,12,20) ( 7,28, 0) (27, 4, 4)
    ( 4,11,20) -> ( 0,15,20) ( 4,28, 3) (15, 0,20) (24,11, 0)
```

```
( 4,28, 3) -> ( 0,28, 7) ( 4,11,20) ( 7,28, 0) (27, 5, 3)
( 5,10,20) -> ( 0,15,20) ( 5,28, 2) (15, 0,20) (25,10, 0)
( 5,28, 2) -> ( 0,28, 7) ( 5,10,20) ( 7,28, 0) (27, 6, 2)
( 6, 9,20) -> ( 0,15,20) ( 6,28, 1) (15, 0,20) (26, 9, 0)
( 6,28, 1) -> ( 0,28, 7) ( 6, 9,20) ( 7,28, 0) (27, 7, 1)
( 7, 8,20) -> ( 0,15,20) ( 7,28, 0) (15, 0,20) (27, 8, 0)
( 7,28, 0) -> ( 0,28, 7) ( 7, 8,20) (27, 8, 0)
( 8, 7,20) -> ( 0,15,20) ( 8,27, 0) (15, 0,20) (27, 7, 1)
( 8,27, 0) -> ( 0,27, 8) ( 7,28, 0) ( 8, 7,20) (27, 8, 0)
( 9, 6,20) -> ( 0,15,20) ( 9,26, 0) (15, 0,20) (27, 6, 2)
( 9,26, 0) -> ( 0,26, 9) ( 7,28, 0) ( 9, 6,20) (27, 8, 0)
(10, 5,20) -> ( 0,15,20) (10,25, 0) (15, 0,20) (27, 5, 3)
(10,25, 0) -> ( 0,25,10) ( 7,28, 0) (10, 5,20) (27, 8, 0)
(11, 4,20) -> ( 0,15,20) (11,24, 0) (15, 0,20) (27, 4, 4)
(11,24, 0) -> ( 0,24,11) ( 7,28, 0) (11, 4,20) (27, 8, 0)
(12, 3,20) -> ( 0,15,20) (12,23, 0) (15, 0,20) (27, 3, 5)
(12,23, 0) -> ( 0,23,12) ( 7,28, 0) (12, 3,20) (27, 8, 0)
(13, 2,20) -> ( 0,15,20) (13,22, 0) (15, 0,20) (27, 2, 6)
(13,22, 0) -> ( 0,22,13) ( 7,28, 0) (13, 2,20) (27, 8, 0)
(14, 1,20) -> ( 0,15,20) (14,21, 0) (15, 0,20) (27, 1, 7)
(14,21, 0) -> ( 0,21,14) ( 7,28, 0) (14, 1,20) (27, 8, 0)
(15, 0,20) -> ( 0,15,20) (15,20, 0) (27, 0, 8)
(15,13, 7) -> ( 0,28, 7) ( 2,13,20) (15, 0,20) (15,20, 0) (22,13, 0) (27, 1, 7)
(15,20, 0) -> ( 0,20,15) ( 7,28, 0) (15, 0,20) (27, 8, 0)
(16, 0,19) -> ( 0,16,19) (15, 0,20) (16,19, 0) (27, 0, 8)
(16,19, 0) -> ( 0,19,16) ( 7,28, 0) (16, 0,19) (27, 8, 0)
(17, 0,18) -> ( 0,17,18) (15, 0,20) (17,18, 0) (27, 0, 8)
(17,18, 0) -> ( 0,18,17) ( 7,28, 0) (17, 0,18) (27, 8, 0)
(18, 0,17) -> ( 0,18,17) (15, 0,20) (18,17, 0) (27, 0, 8)
(18,17, 0) -> ( 0,17,18) ( 7,28, 0) (18, 0,17) (27, 8, 0)
(19, 0,16) -> ( 0,19,16) (15, 0,20) (19,16, 0) (27, 0, 8)
(19,16, 0) -> ( 0,16,19) ( 7,28, 0) (19, 0,16) (27, 8, 0)
(20, 0,15) -> ( 0,20,15) (15, 0,20) (20,15, 0) (27, 0, 8)
(20,15, 0) -> ( 0,15,20) ( 7,28, 0) (20, 0,15) (27, 8, 0)
(21, 0,14) -> ( 0,21,14) (15, 0,20) (21,14, 0) (27, 0, 8)
(21,14, 0) -> ( 1,14,20) ( 7,28, 0) (21, 0,14) (27, 8, 0)
(22, 0,13) -> ( 0,22,13) (15, 0,20) (22,13, 0) (27, 0, 8)
(22,13, 0) -> ( 2,13,20) ( 7,28, 0) (22, 0,13) (27, 8, 0)
(23, 0,12) -> ( 0,23,12) (15, 0,20) (23,12, 0) (27, 0, 8)
(23,12, 0) -> ( 3,12,20) ( 7,28, 0) (23, 0,12) (27, 8, 0)
(24, 0,11) -> ( 0,24,11) (15, 0,20) (24,11, 0) (27, 0, 8)
(24,11, 0) -> ( 4,11,20) ( 7,28, 0) (24, 0,11) (27, 8, 0)
(25, 0,10) -> ( 0,25,10) (15, 0,20) (25,10, 0) (27, 0, 8)
(25,10, 0) -> ( 5,10,20) ( 7,28, 0) (25, 0,10) (27, 8, 0)
(26, 0, 9) -> ( 0,26, 9) (15, 0,20) (26, 9, 0) (27, 0, 8)
(26, 9, 0) -> ( 6, 9,20) ( 7,28, 0) (26, 0, 9) (27, 8, 0)
(27, 0, 8) -> ( 0,27, 8) (15, 0,20) (27, 8, 0)
(27, 1, 7) -> ( 0,28, 7) (14, 1,20) (27, 0, 8) (27, 8, 0)
(27, 2, 6) -> ( 1,28, 6) (13, 2,20) (27, 0, 8) (27, 8, 0)
(27, 3, 5) -> ( 2,28, 5) (12, 3,20) (27, 0, 8) (27, 8, 0)
(27, 4, 4) -> ( 3,28, 4) (11, 4,20) (27, 0, 8) (27, 8, 0)
(27, 5, 3) -> ( 4,28, 3) (10, 5,20) (27, 0, 8) (27, 8, 0)
(27, 6, 2) -> ( 5,28, 2) ( 9, 6,20) (27, 0, 8) (27, 8, 0)
(27, 7, 1) -> ( 6,28, 1) ( 8, 7,20) (27, 0, 8) (27, 8, 0)
(27, 8, 0) -> ( 7, 8,20) ( 7,28, 0) (27, 0, 8)

+++ Make the following moves
    (15,13, 7) ==> Transfer 13 ml from Glass 2 to Glass 3 ==>
    (15, 0,20) ==> Transfer 12 ml from Glass 3 to Glass 1 ==>
    (27, 0, 8) ==> Transfer 27 ml from Glass 1 to Glass 2 ==>
    ( 0,27, 8) ==> Transfer  8 ml from Glass 3 to Glass 1 ==>
    ( 8,27, 0) ==> Transfer 20 ml from Glass 2 to Glass 3 ==>
    ( 8, 7,20) ==> Transfer 19 ml from Glass 3 to Glass 1 ==>
    (27, 7, 1) ==> Transfer 21 ml from Glass 1 to Glass 2 ==>
    ( 6,28, 1) ==> Transfer 19 ml from Glass 2 to Glass 3 ==>
    ( 6, 9,20) ==> Transfer 20 ml from Glass 3 to Glass 1 ==>
    (26, 9, 0) ==> Transfer  9 ml from Glass 2 to Glass 3 ==>
    (26, 0, 9) ==> Transfer 26 ml from Glass 1 to Glass 2 ==>
    ( 0,26, 9) ==> Transfer  9 ml from Glass 3 to Glass 1 ==>
    ( 9,26, 0) ==> Transfer 20 ml from Glass 2 to Glass 3 ==>
    ( 9, 6,20) ==> Transfer 18 ml from Glass 3 to Glass 1 ==>
    (27, 6, 2) ==> Transfer 22 ml from Glass 1 to Glass 2 ==>
    ( 5,28, 2) ==> Transfer 18 ml from Glass 2 to Glass 3 ==>
    ( 5,10,20)
```

Write a single C/C++ source file. Do not use global/static variables.