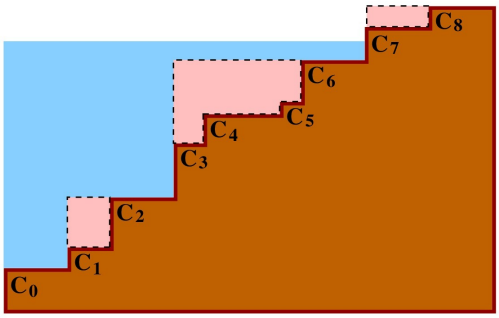
CS29003 ALGORITHMS LABORATORY
Assignment No: 10
Last Date of Submission: 16-October-2014

Imagine a situation as depicted in the following figure. A dam is built on the river Sweta (water is shown as blue). A vertical cross section of the dam is shown as the brown area. The dam has n steps, each with a known width and height (in the figure $n = 9$). At every step, there is a controller (shown as C_i). Once upon a time, it is discovered that some of the controllers are faulty and beyond repair. Only m of the n controllers are functioning properly. The controllers may be relocated, but for these to work properly, we have to merge some steps. The figure describes three cases of merging in pink color: C_1 is merged with C_2, and C_7 with C_8. Multiple consecutive steps may also be merged, as demonstrated by the merging of C_3, C_4, C_5 and C_6. After these merges, only four functioning controllers suffice (so $m = 4$ in this example).

Merging involves filling the steps with concrete. This is possible only during the driest season which is short-lived and is followed by torrential rains in which restructuring efforts cannot proceed. Therefore, it is needed to find the merging possibility (from n to m steps/controllers) such that the merging area (the total pink area) is as small as possible. Your task is to devise a dynamic-programming algorithm to solve this problem.



First, read n and m from the user. Then, the user enters the widths and heights of the n steps. Write and call a function to store in a two-dimensional array the cost of merging controllers i through j for all pairs (i, j). Then, write a function to compute the minimum cost of replacing n controllers by m controllers using a dynamic-programming approach. More *concrete*ly, build a two-dimensional table T, in which the (i, j)-th cell stores the optimal cost of replacing the controllers C_0 to C_i by the controllers C_0 to C_j. Optimize over where you put C_j. Modify this function so as to compute the optimal way of merging steps. Print the optimal cost and the optimal solution.

Sample Output

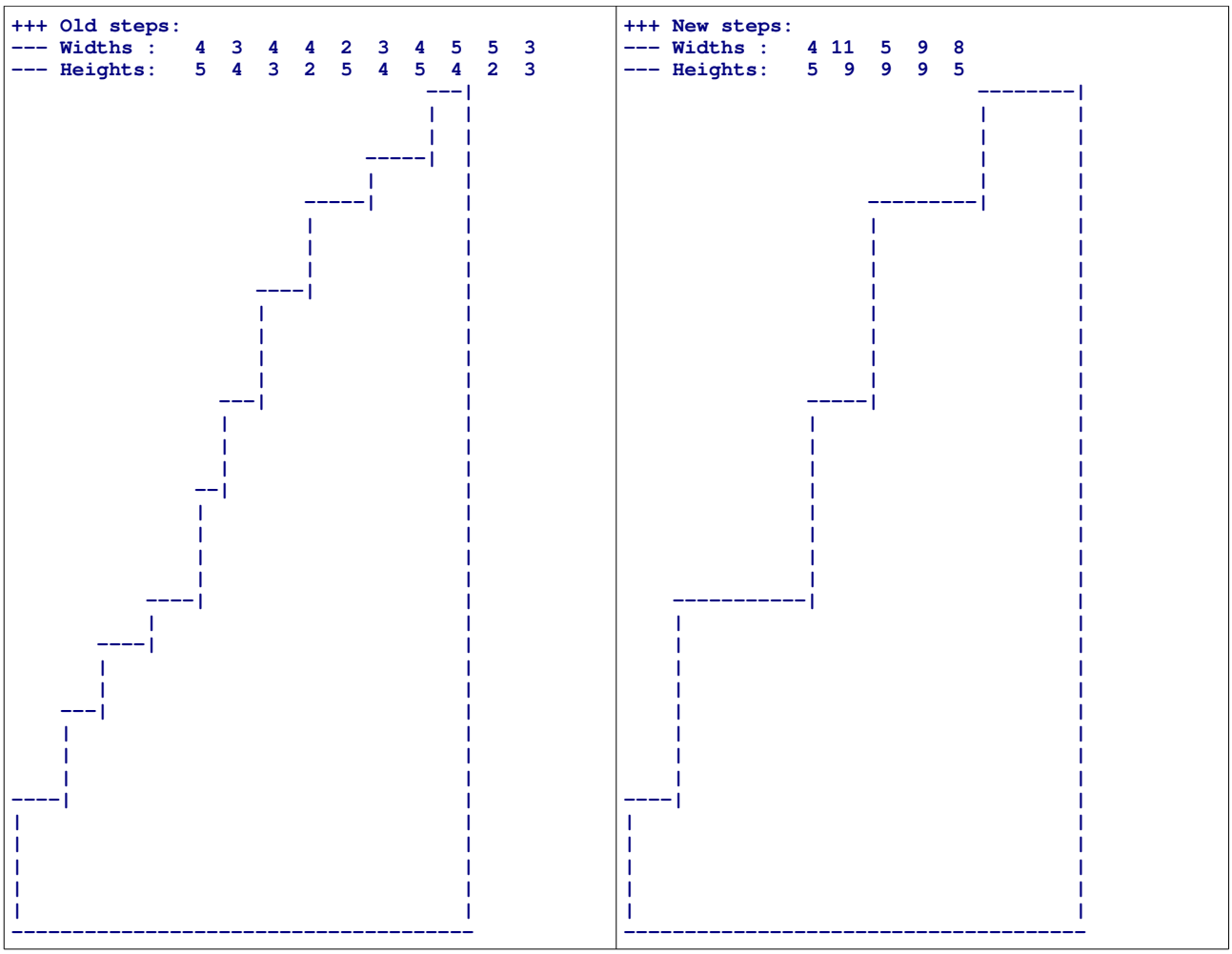
The following transcript demonstrates replacing 10 controllers by 5 controllers.

```
+++ Old steps:
--- Widths :   4  3  4  4  2  3  4  5  5  3
--- Heights:   5  4  3  2  5  4  5  4  2  3

Minimum restructuring cost = 62

+++ New steps:
--- Widths :   4 11  5  9  8
--- Heights:   5  9  9  9  5
```

A graphical representation of the old and the new steps is shown on the next page. If time permits, you may implement this. This example demonstrates that C_0 is not merged, C_1, C_2 and C_3 are merged, and so also are the pairs (C_4,C_5), (C_6,C_7) and (C_8,C_9).


+++ Old steps:
--- Widths : 4 3 4 4 2 3 4 5 5 3
--- Heights: 5 4 3 2 5 4 5 4 2 3
+++ New steps:
--- Widths : 4 11 5 9 8
--- Heights: 5 9 9 9 5