Public-key Cryptography
Theory and Practice

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Chapter 5: Cryptographic Algorithms

Common Encryption Algorithms

Encryption algorithm	Security depends on
RSA encryption	Integer factoring problem
ElGamal encryption	DHP (DLP)
Rabin encryption	Square-root problem
Goldwasser-Micali encryption	Quadratic residuosity problem
Blum-Goldwasser encryption	Square-root problem
Chor-Rivest encryption	Subset sum problem
XTR	DLP
NTRU	Closest vector problem in lattices

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- **Key generation**
  The recipient generates two random large primes p, q, computes $n = pq$ and $\phi(n) = (p-1)(q-1)$, finds a random integer e with $\gcd(e, \phi(n)) = 1$, and determines an integer d with $ed \equiv 1 \pmod{\phi(n)}$.
  Public key: (n, e).
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  Input: Plaintext $m \in \mathbb{Z}_n$ and the recipient's public key (n, e).
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  Public key: (n, e).
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- **Encryption**
  Input: Plaintext $m \in \mathbb{Z}_n$ and the recipient's public key (n, e).
  Output: Ciphertext $c \equiv m^e \pmod{n}$.
- **Decryption**
  Input: Ciphertext c and the recipient's private key (n, d).
  Output: Plaintext $m \equiv c^d \pmod{n}$.

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- Let $p = 257$, $q = 331$, so that $n = pq = 85067$ and $\phi(n) = (p-1)(q-1) = 84480$. Take $e = 7$, so that $d \equiv e^{-1} \equiv 60343 \pmod{\phi(n)}$.

  Public key: $(85067, 7)$.

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- Let $m = 34152$. Then $c \equiv m^e \equiv (34152)^7 \equiv 53384 \pmod{n}$.

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- Recover $m \equiv c^d \equiv (53384)^{60343} \equiv 34152 \pmod{n}$.

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  Public key: $(85067, 7)$.
  Private key: $(85067, 60343)$.
- Let $m = 34152$. Then $c \equiv m^e \equiv (34152)^7 \equiv 53384 \pmod{n}$.
- Recover $m \equiv c^d \equiv (53384)^{60343} \equiv 34152 \pmod{n}$.
- Decryption by an exponent d' other than d does not give back m. For example, take $d' = 38367$. We have $m' \equiv c^{d'} \equiv (53384)^{38367} \equiv 71303 \pmod{n}$.

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- Now, $ed \equiv 1 \pmod{\phi(n)}$, that is, $ed = 1 + k\phi(n)$ for some integer k. Therefore,

$$c^d \equiv m^{ed} \equiv m^{1+k\phi(n)} \equiv m \times \left(m^{\phi(n)}\right)^k \equiv m \times 1^k \equiv m \pmod{n}.$$

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- **Note:** The message can be recovered uniquely even when $m \notin \mathbb{Z}_n^*$.

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- If e, d, n are known, there exists a probabilistic polynomial-time algorithm to factor n. So RSA key inversion is as difficult as IFP. But RSA decryption without the knowledge of d *may be* easier than factoring n.
- In practice, we require the size of n to be $\geqslant 1024$ bits with each of p, q having nearly half the size of n.

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- Store $n, e, d, p, q, d_1, d_2, h$, where $d_1 = d \text{ rem } (p-1)$, $d_2 = d \text{ rem } (q-1)$ and $h = q^{-1} \pmod{p}$.

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- Carry out decryption as:
  - $m_1 = c^{d_1} \pmod{p}$.
  - $m_2 = c^{d_2} \pmod{q}$.
  - $t = h(m_1 - m_2) \pmod{p}$.
  - $m = m_2 + tq$.

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  - $m = m_2 + tq$.
- A speedup of about 4 is obtained.

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- **Key generation**
  The recipient selects a random big prime p and a primitive root g modulo p, chooses a random $d \in \{2, 3, \ldots, p-2\}$, and computes $y \equiv g^d \pmod{p}$.
  Public key: (p, g, y).
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- **Encryption**
  Input: Plaintext $m \in \mathbb{Z}_p$ and recipient's public key (p, g, y).
  Output: Ciphertext (s, t).
  Generate a random integer $d' \in \{2, 3, \ldots, p-2\}$.
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- **Decryption**
  Input: Ciphertext (s, t) and recipient's private key (p, g, d).
  Output: Recovered plaintext $m \equiv ts^{-d} \pmod{p}$.

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  - Take $p = 91573$ and $g = 67$. The recipient chooses $d = 23632$ and so $y \equiv (67)^{23632} \equiv 87955 \pmod{p}$.

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  - Let $m = 29485$ be the message to be encrypted. The sender chooses $d' = 1783$ and computes $s \equiv g^{d'} \equiv 52958 \pmod{p}$ and $t \equiv my^{d'} \equiv 1597 \pmod{p}$.

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  - Let $m = 29485$ be the message to be encrypted. The sender chooses $d' = 1783$ and computes $s \equiv g^{d'} \equiv 52958 \pmod{p}$ and $t \equiv my^{d'} \equiv 1597 \pmod{p}$.
  - The recipient retrieves $m \equiv ts^{-d} \equiv 1597 \times (52958)^{-23632} \equiv 29485 \pmod{p}$.

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- An eavesdropper knows g, p, y, s, t, where $y \equiv g^d \pmod{p}$ and $s \equiv g^{d'} \pmod{p}$. Determining m from (s, t) is equivalent to computing $g^{dd'} \pmod{p}$, since $t \equiv m g^{dd'} \pmod{p}$. (Here, m is masked by the quantity $g^{dd'} \pmod{p}$.) But d, d' are unknown to the attacker. So the ability to solve the DHP lets the eavesdropper break ElGamal encryption.

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- Practically, we require p to be of size $\geqslant 1024$ bits for achieving a good level of security.

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- **Quadratic residuosity problem:** For a composite integer n and for an a with $\left(\frac{a}{n}\right) = 1$, determine whether a is a quadratic residue modulo n, that is, the whether the congruence $x^2 \equiv a \pmod{n}$ is solvable.

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- Suppose $n = pq$ (product of two primes). $\left(\frac{a}{n}\right) = 1$ implies either $\left(\frac{a}{p}\right) = \left(\frac{a}{q}\right) = 1$ (a is a quadratic residue) or $\left(\frac{a}{p}\right) = \left(\frac{a}{q}\right) = -1$ (a is a quadratic non-residue).

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- We know no methods other than factoring n to solve this problem.

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- Use CRT to generate $x \pmod{n}$ with $x \equiv a \pmod{p}$ and $x \equiv b \pmod{q}$.
- The Public key is (n, x), and the private key is p.

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- For each $i = 1, 2, \ldots, r$, choose $a_i \in \mathbb{Z}_n^*$ randomly and compute $c_i = x^{m_i} a_i^2 \pmod{n}$.

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Decryption

- For $i = 1, 2, \ldots, r$, take

$$m_i = 0 \text{ if } \left(\frac{c_i}{p}\right) = 1, \text{ or}$$

$$m_i = 1 \text{ if } \left(\frac{c_i}{p}\right) = -1.$$

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- **Probabilistic encryption:** The ciphertext c_i depends on the choice of a_i.
- **Message expansion:** An r-bit plaintext message generates an rl-bit ciphertext message, where $l = |n|$.
- Without the knowledge of p (the private key), we do not know how to determine whether c_i is a quadratic residue or not modulo n.

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- Take $p = 653$ and $q = 751$, so $n = pq = 490403$.
- Take $a = 159$ and $b = 432$, so $x \equiv 313599 \pmod{n}$.

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- Choose $a_3 = 482474$ and compute $c_3 \equiv x a_3^2 \equiv 12380 \pmod{n}$.

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- Let us encrypt the 3-bit message $m_1 m_2 m_3 = 101$.
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- Choose $a_3 = 482474$ and compute $c_3 \equiv x a_3^2 \equiv 12380 \pmod{n}$.

Decryption

Goldwasser-Micali Encryption: Example

Key generation

- Take $p = 653$ and $q = 751$, so $n = pq = 490403$.
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- The Diffie-Hellman protocol works in other groups (finite extension fields and elliptic curve groups).

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- Breaking DH key exchange *may be* easier than solving DLP.
- At present, no method other than computing discrete logs in $\mathbb{Z}_p^*$ is known to break DH key exchange.
- Practically, we require p to be of size $\geqslant 1024$ bits. The security does not depend on the choice of g. However, a and b must be sufficiently randomly chosen.

Common Signature Algorithms

Signature algorithm	Security depends on
RSA signature	Integer factoring problem
ElGamal encryption	DLP
Rabin signature	Square-root problem
Schnorr signature	DLP
Nyberg-Rueppel signature	DLP
Digital signature algorithm (DSA)	DLP
Elliptic curve version of DSA (ECDSA)	DLP in elliptic curves
XTR signature	DLP
NTRUSign	Closest vector problem

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  - **Multiple-use signatures:** Slow. Parameters are used multiple times.
  - **One-time signatures:** Fast. Parameters are used only once.

RSA Signature

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- **Key generation**
  The signer generates two random large primes p, q, computes $n = pq$ and $\phi(n) = (p-1)(q-1)$, finds a random integer e with $\gcd(e, \phi(n)) = 1$, and determines an integer d with $ed \equiv 1 \pmod{\phi(n)}$.

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- **Signature verification**
  Input: Signed message (m, s) and signer's public key (n, e).
  Output: "Signature verified" if $s^e \equiv m \pmod{n}$,
  "Signature not verified" if $s^e \not\equiv m \pmod{n}$.

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- Let $p = 257$, $q = 331$, so that $m = pq = 85067$ and $\phi(n) = (p-1)(q-1) = 84480$. Take $e = 19823$, so that $d \equiv e^{-1} \equiv 71567 \pmod{\phi(n)}$.
  Public key: $(85067, 19823)$.
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- Let $m = 3759$ be the message to be signed. Generate $s \equiv m^d \equiv 13728 \pmod{n}$. The signed message is $(3759, 13728)$.

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- Verification of a forged signature $(m, s) = (3759, 42954)$ gives $s^e \equiv (42954)^{19823} \equiv 22968 \pmod{n}$. Since $s^e \not\equiv m \pmod{n}$, the forged signature is not verified.

ElGamal Signature

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- **Key generation**
  Like ElGamal encryption, one chooses p, g and computes a key-pair (y, d) where $y \equiv g^d \pmod{p}$. The public key is (p, g, y), and the private key is (p, g, d).

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- **Signature generation**
  Input: Message $m \in \mathbb{Z}_p$ and signer's private key (p, g, d).
  Output: Signed message (m, s, t).
  Generate a random session key $d' \in \{2, 3, \ldots, p-2\}$.
  Compute $s \equiv g^{d'} \pmod{p}$ and
  $t \equiv d'^{-1}(H(m) - dH(s)) \pmod{p-1}$.

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- **Signature verification**
  Input: Signed message (m, s, t) and signer's public key (p, g, y).
  Set $a_1 \equiv g^{H(m)} \pmod{p}$ and $a_2 \equiv y^{H(s)} s^t \pmod{p}$.
  Output "signature verified" if and only if $a_1 = a_2$.

ElGamal Signature (contd)

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- **Correctness:** $H(m) \equiv dH(s) + td' \pmod{p-1}$. So $a_1 \equiv g^{H(m)} \equiv (g^d)^{H(s)}(g^{d'})^t \equiv y^{H(s)}s^t \equiv a_2 \pmod{p}$.

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  - Take $p = 104729$ and $g = 89$. The signer chooses the private exponent $d = 72135$ and so $y \equiv g^d \equiv 98771 \pmod{p}$.

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  - Take $p = 104729$ and $g = 89$. The signer chooses the private exponent $d = 72135$ and so $y \equiv g^d \equiv 98771 \pmod{p}$.
  - Let $m = 23456$ be the message to be signed. The signer chooses the session exponent $d' = 3951$ and computes $s \equiv g^{d'} \equiv 14413 \pmod{p}$ and $t \equiv d'^{-1}(m - ds) \equiv (3951)^{-1}(23456 - 72135 \times 14413) \equiv 17515 \pmod{p-1}$.

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  - Verification involves computation of $a_1 \equiv g^m \equiv 29201 \pmod{p}$ and $a_2 \equiv y^s s^t \equiv (98771)^{14413} \times (14413)^{17515} \equiv 29201 \pmod{p}$. Since $a_1 = a_2$, the signature is verified.

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- **Forging:** A forger chooses $d' = 3951$ and computes $s \equiv g^{d'} \equiv 14413 \pmod{p}$. But computation of t involves d which is unknown to the forger. So the forger randomly selects $t = 81529$. Verification of this forged signature gives $a_1 \equiv g^m \equiv 29201 \pmod{p}$ as above. But $a_2 \equiv y^s s^t \equiv (98771)^{14413} \times (14413)^{81529} \equiv 85885 \pmod{p}$, that is, $a_1 \neq a_2$ and the forged signature is not verified.

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  - The prime p should be large (of bit-size $\geqslant 1024$) in order to preclude this attack.

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Accepted by the US Government as a standard.

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Digital Signature Algorithm (DSA)

Accepted by the US Government as a standard.

Parameter generation

- Generate a prime p of bit length $512 + 64\lambda$ for $0 \leqslant \lambda \leqslant 8$.
- $p - 1$ must have a prime divisor r of bit length 160.
- A specific algorithm is recommended for computing p and r.
- Compute an element $g \in \mathbb{F}_p^*$ with multiplicative order r.
- Make p, r, g public.

Key generation

- Generate a random $d \in \{2, 3, \ldots, r-1\}$ (private key).
- Compute $y \equiv g^d \pmod{p}$ (public key).

DSA (contd)

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Signature generation

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- Compute $s = \left(g^{d'} \pmod p\right) \pmod r$ and $t = d'^{-1}(H(M) + ds) \pmod r$.

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- Output the signed message (M, s, t).

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- To verify a signature (M, s, t) using the signer's public key y:

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- Compute $\tilde{s} = (g^{w_1} y^{w_2} \pmod p) \pmod r$.

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- Compute $\tilde{s} = (g^{w_1} y^{w_2} \pmod p) \pmod r$.
- Signature is verified if and only if $\tilde{s} = s$.

DSA (contd)

Correctness

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Remarks

- Although the modulus p may be as long as 1024 bits, the signature size (s, t) is only 320 bits.

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Remarks

- Although the modulus p may be as long as 1024 bits, the signature size (s, t) is only 320 bits.
- The security of DSA depends on the difficulty of solving the DLP in $\mathbb{F}_p^*$.

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Parameters:

$p = 21101$, $r = 211$, and $g \equiv 12345^{(p-1)/r} \equiv 17808 \pmod{p}$.

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Signature generation:

- To sign $M = 8642$.

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- Choose $d' = 167$.

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Key pair:

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Signature generation:

- To sign $M = 8642$.
- Choose $d' = 167$.
- Compute $s = \left(g^{d'} \pmod{p}\right) \pmod{r} = 13687 \text{ rem } r = 183$.

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Key pair:

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Signature generation:

- To sign $M = 8642$.
- Choose $d' = 167$.
- Compute $s = \left(g^{d'} \pmod{p}\right) \pmod{r} = 13687 \text{ rem } r = 183$.
- Compute $t \equiv d'^{-1}(M + ds) \equiv 132 \pmod{r}$.

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Signature generation:

- To sign $M = 8642$.
- Choose $d' = 167$.
- Compute $s = \left(g^{d'} \pmod{p}\right) \pmod{r} = 13687 \text{ rem } r = 183$.
- Compute $t \equiv d'^{-1}(M + ds) \equiv 132 \pmod{r}$.
- The signature is the pair $(183, 132)$.

DSA: Example (contd)

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Signature verification:

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- To verify $(8642, 183, 132)$.

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Signature verification:

- To verify $(8642, 183, 132)$.
- Compute $w \equiv t^{-1} \equiv 8 \pmod{r}$.

DSA: Example (contd)

Signature verification:

- To verify $(8642, 183, 132)$.
- Compute $w \equiv t^{-1} \equiv 8 \pmod{r}$.
- Compute $w_1 \equiv Mw \equiv 8642 \times 8 \equiv 139 \pmod{r}$.

DSA: Example (contd)

Signature verification:

- To verify $(8642, 183, 132)$.
- Compute $w \equiv t^{-1} \equiv 8 \pmod{r}$.
- Compute $w_1 \equiv Mw \equiv 8642 \times 8 \equiv 139 \pmod{r}$.
- Compute $w_2 \equiv sw \equiv 183 \times 8 \equiv 198 \pmod{r}$.

DSA: Example (contd)

Signature verification:

- To verify $(8642, 183, 132)$.
- Compute $w \equiv t^{-1} \equiv 8 \pmod{r}$.
- Compute $w_1 \equiv Mw \equiv 8642 \times 8 \equiv 139 \pmod{r}$.
- Compute $w_2 \equiv sw \equiv 183 \times 8 \equiv 198 \pmod{r}$.
- $g^{w_1} y^{w_2} \equiv 17808^{139} \times 2377^{198} \equiv 13687 \pmod{p}$.

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- Compute $w_1 \equiv Mw \equiv 8642 \times 8 \equiv 139 \pmod{r}$.
- Compute $w_2 \equiv sw \equiv 183 \times 8 \equiv 198 \pmod{r}$.
- $g^{w_1} y^{w_2} \equiv 17808^{139} \times 2377^{198} \equiv 13687 \pmod{p}$.
- $\tilde{s} = 13687 \text{ rem } r = 183 = s$, so signature is verified.

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- $g^{w_1} y^{w_2} \equiv 17808^{139} \times 2377^{198} \equiv 13687 \pmod{p}$.
- $\tilde{s} = 13687 \text{ rem } r = 183 = s$, so signature is verified.

Verification of faulty signature:

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- $\tilde{s} = 13687 \text{ rem } r = 183 = s$, so signature is verified.

Verification of faulty signature:

- To verify $(8642, 138, 123)$.

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Signature verification:

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- $g^{w_1} y^{w_2} \equiv 17808^{139} \times 2377^{198} \equiv 13687 \pmod{p}$.
- $\tilde{s} = 13687 \text{ rem } r = 183 = s$, so signature is verified.

Verification of faulty signature:

- To verify $(8642, 138, 123)$.
- Compute $w \equiv t^{-1} \equiv 199 \pmod{r}$.

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- $g^{w_1} y^{w_2} \equiv 17808^{139} \times 2377^{198} \equiv 13687 \pmod{p}$.
- $\tilde{s} = 13687 \text{ rem } r = 183 = s$, so signature is verified.

Verification of faulty signature:

- To verify $(8642, 138, 123)$.
- Compute $w \equiv t^{-1} \equiv 199 \pmod{r}$.
- Compute $w_1 \equiv Mw \equiv 8642 \times 199 \equiv 108 \pmod{r}$.

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- $g^{w_1} y^{w_2} \equiv 17808^{108} \times 2377^{32} \equiv 3838 \pmod{p}$.

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- $g^{w_1} y^{w_2} \equiv 17808^{108} \times 2377^{32} \equiv 3838 \pmod{p}$.
- $\tilde{s} = 3838 \text{ rem } r = 40 \neq s$, so signature is not verified.

Elliptic Curve Digital Signature Algorithm (ECDSA)

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1. Choose $a, b \in \mathbb{F}_q$ randomly.
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7. Choose $P' \in E(\mathbb{F}_q)$ randomly.
8. Compute $P = (n/r)P'$.

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7. Choose $P' \in E(\mathbb{F}_q)$ randomly.
8. Compute $P = (n/r)P'$.
9. If $P = \mathcal{O}$, go to Step 7.

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7. Choose $P' \in E(\mathbb{F}_q)$ randomly.
8. Compute $P = (n/r)P'$.
9. If $P = \mathcal{O}$, go to Step 7.
10. Return E, n, r, P.

ECDSA (contd)

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- Choose $d \in \{2, 3, \ldots, r-1\}$ randomly [private key].

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- Choose $d \in \{2, 3, \ldots, r-1\}$ randomly [private key].
- Compute $Y = dP \in E(\mathbb{F}_q)$ [public key].

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Signature generation: (To sign message M)

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Signature generation: (To sign message M)

- Choose session key $d' \in \{2, 3, \ldots, r-1\}$.

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- Choose $d \in \{2, 3, \ldots, r-1\}$ randomly [private key].
- Compute $Y = dP \in E(\mathbb{F}_q)$ [public key].

Signature generation: (To sign message M)

- Choose session key $d' \in \{2, 3, \ldots, r-1\}$.
- Compute $(h, k) = d'P \in E(\mathbb{F}_q)$.

ECDSA (contd)

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Examples

Chaum-van Antwerpen undeniable signature scheme

RSA-based undeniable signature scheme

Entity Authentication

Weak Authentication: Passwords

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- If $Q' = Q$, Bob accepts Alice's identity.

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- P resides in unencrypted form in the memory during the authentication phase. A third party having access to this memory obtains Alice's secret.

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- This conversation does not reveal any information about the secret to the verifier or to an eavesdropper.

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- Use of r_B prevents **replay attacks** by an eavesdropper.
- Use of **timestamps** achieves the same objective. Alice signs $s = f_d(t_A, d)$ and sends (t_A, s) to Bob. Bob retrieves $t'_A = f_e(s, e)$. Bob accepts Alice if and only if $t_A = t'_A$ and t_A is a valid timestamp.

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- The protocol may be repeated multiple times (t times), so that the probability of success for an eavesdropper (P^t) can be made as small as desirable.

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- Make $y_1, y_2, \ldots, y_t$ public. Keep $x_1, x_2, \ldots, x_t$ secret.

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- **[Authentication]** Bob computes $w' \equiv r^2 \prod_{i=1}^{t} y_i^{\epsilon_i} \pmod{n}$, and accepts Alice's identity if and only if $w' \neq 0$ and $w' \equiv \pm w \pmod{n}$.

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- Consequently, $w' \equiv \pm w \pmod{n}$.
- The check $w' \neq 0$ eliminates the commitment $c = 0$ which succeeds always irrespective of the knowledge of the secrets x_i.

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- In either case, the success probability is nearly $1/2$.

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- If signatures cannot be forged, nobody other than the CA can generate a valid certificate for an entity.

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- **Certificate Revocation List** (**CRL**): The CA maintains a list of revoked certificates.
- If Alice wants to use Bob's public key, she obtains the certificate for Bob's public key. If the CA's signature is verified on this certificate and if the certificate is not found in the CRL, then Alice gains the desired confidence to use Bob's public key.