

# Public-key Cryptography

## Theory and Practice

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### Chapter 1: Overview

# What is Cryptography?

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- Maintaining security and privacy is an ancient and primitive need.
- Particularly relevant for military and diplomatic applications.
- Wide deployment of the Internet makes everybody a user of cryptographic tools.

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- Knowledge of  $K_d$  is required to retrieve  $M$  from  $C$ .
- An eavesdropper (intruder, attacker, adversary, opponent, enemy) cannot decrypt  $C$ .

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- If there are many communicating pairs, the key storage requirement is high.

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- Anybody can send messages to anybody. Only the proper recipient can decrypt.
- No need to establish keys a priori.
- Each party requires only one key-pair for communicating with everybody.
- Algorithms are slow, in general.

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## Asymmetric encryption

- Alice presses a self-locking padlock in order to lock the box. The locking process does not require a real key.
- Bob has the key to open the padlock.

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- Alice sends  $(C, L)$  to Bob.

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- Alice encrypts  $K$  by  $K_e$  to generate  $L = f_E(K, K_e)$ .
- Alice sends  $(C, L)$  to Bob.
- Bob recovers  $K$  as  $K = f_D(L, K_d)$ .

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- Bob decrypts  $C$  as  $M = f_d(C, K)$ .

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- Bob locks the box by  $L_B$  using  $K_B$ , and sends the doubly-locked box back to Alice.

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- Bob unlocks  $L_B$  by  $K_B$  and obtains  $K$ .
- A third party always finds the box locked either by  $L_A$  or  $L_B$  or both.

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- Alice computes  $K_{AB} = f(A_e, A_d, B_e)$ .

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- The protocol insures  $K_{AB} = K_{BA}$  to be used by Alice and Bob as a shared secret.
- An intruder cannot compute this secret using  $A_e$  and  $B_e$  only.



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- Signature generation  $\equiv$  Decryption (uses private key), and Signature verification  $\equiv$  Encryption (uses public key).
- **Non-repudiation:** An entity should not be allowed to deny valid signatures made by him.



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## Drawback

- Algorithms are slow, not suitable for long messages.

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- Compute the representative  $m = H(M)$ .
- Use Alice's public-key to generate  $m' = f_v(s, K_e)$ .
- Accept the signature if and only if  $m = m'$ .

# Signature With Appendix

## Generation

- Alice generates a key-pair  $(K_e, K_d)$ , publishes  $K_e$ , and keeps  $K_d$  secret.
- Alice generates a short representative  $m = H(M)$  of  $M$ .
- Alice uses her private-key:  $s = f_s(m, K_d)$ .
- Alice publishes  $(M, s)$  as the signed message.

## Verification

- Compute the representative  $m = H(M)$ .
- Use Alice's public-key to generate  $m' = f_v(s, K_e)$ .
- Accept the signature if and only if  $m = m'$ .

## Forging

- Verification is expected to fail if a key  $K'_d \neq K_d$  is used to generate  $s$ .

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## Drawback

- $C$  and  $R$  may reveal to Bob or an eavesdropper some knowledge about Alice's secret.

# Zero-knowledge Protocol

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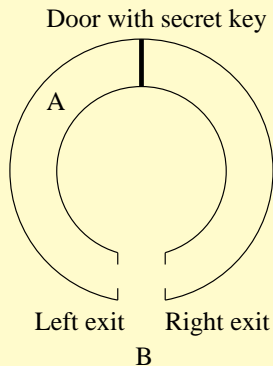
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## A real-life example



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- If a certificate is not in the CRL, and the signature of the CA on the certificate is verified, one gains the desired confidence of treating the public-key as authentic.

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- **Active attack**

The attacker alters and/or deletes messages and even creates unauthorized messages.

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