Public-key Cryptography
Theory and Practice

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Appendix A: Symmetric Techniques

Block Ciphers

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- A block cipher f of **block-size** n and **key-size** r is a function

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  taking a plaintext message M to the ciphertext message $C = f_K(M) = f(M, K)$ should be bijective (invertible).
- n and r should be large enough to preclude successful exhaustive search.
- Each f_K should be a sufficiently random permutation.

Block Ciphers: Examples

Name	n, r
DES (Data Encryption Standard)	64, 56
FEAL (Fast Data Encipherment Algorithm)	64, 64
SAFER (Secure And Fast Encryption Routine)	64, 64
IDEA (International Data Encryption Algorithm)	64, 128
Blowfish	64, $\leqslant$ 448
The AES Finalists	
Rijndael (Rijmen and Daemen)	128, 128/192/256
Serpent (Anderson, Biham and Knudsen)	128, 128/192/256
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Old standard: DES

New standard: AES (adaptation of the Rijndael cipher)

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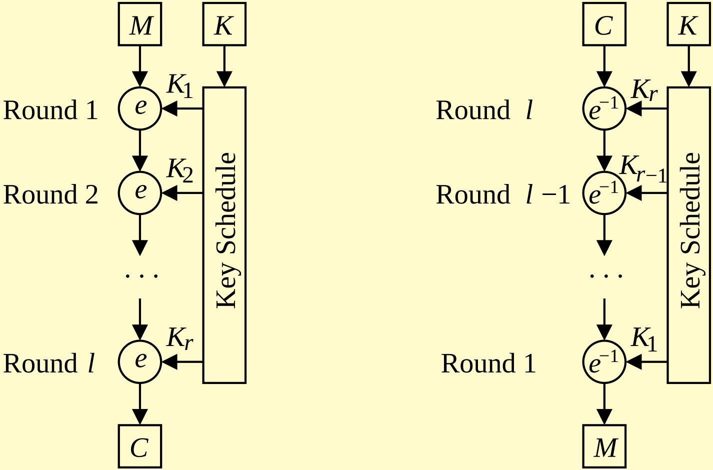
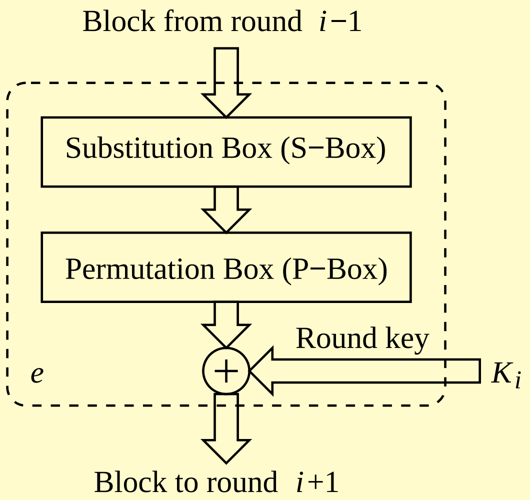
  - Diffusion is meant to dissipate plaintext redundancy.

Iterated Block Cipher

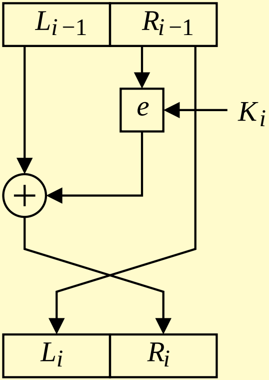


(a) Encryption (b) Decryption

Substitution-Permutation Network (SPN)



Feistel Cipher



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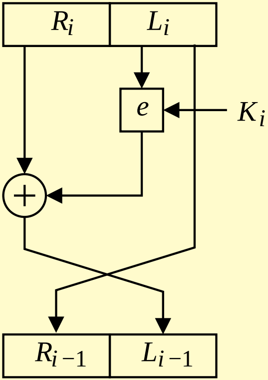

(b) Decryption

$$\begin{aligned} L_i &= R_{i-1} \\ R_i &= L_{i-1} \oplus e(R_{i-1}, K_i) \end{aligned}$$

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- DES is a Feistel cipher.
- The number of rounds in DES is $l = 16$.

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  - Let $U_i := C_i \,||\, D_i = u_1 u_2 \ldots u_{56}$.
  - Compute 48-bit round key $K_i = \mathrm{PC2}(U_i) = u_{14} u_{17} u_{11} \ldots u_{29} u_{32}$.

DES Key Schedule (contd)

PC1

57	49	41	33	25	17	9
1	58	50	42	34	26	18
10	2	59	51	43	35	27
19	11	3	60	52	44	36
63	55	47	39	31	23	15
7	62	54	46	38	30	22
14	6	61	53	45	37	29
21	13	5	28	20	12	4

PC2

14	17	11	24	1	5
3	28	15	6	21	10
23	19	12	4	26	8
16	7	27	20	13	2
41	52	31	37	47	55
30	40	51	45	33	48
44	49	39	56	34	53
46	42	50	36	29	32

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- Let $W = R_{16} \,||\, L_{16} = w_1 w_2 \ldots w_{64}$.
- Apply inverse of IP: $C = \text{IP}^{-1}(W) = w_{40} w_8 w_{48} \ldots w_{57} w_{25}$.

DES Encryption (contd)

IP

58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

IP^{-1}

40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

DES Encryption (contd)

Encryption primitive

$$e(X, J) = \mathrm{P}(S(\mathrm{E}(X) \oplus J)),$$

where X is a 32-bit message block, and J is a 48-bit round key.

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- Apply 32-to-48 bit expansion: $X' = \mathrm{E}(X) = x_{32}x_1x_2 \ldots x_{32}x_1$.

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- XOR with the round key: $Y = X' \oplus J$.
- Break Y in eight 6-bit parts: $Y = Y_1 \mid\mid Y_2 \mid\mid \cdots \mid\mid Y_8$.

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- For $j = 1, 2, \ldots, 8$, do the following:

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  - Consider the integers $\mu = (y_1y_6)_2$ and $\nu = (y_2y_3y_4y_5)_2$.

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  - Consider the integers $\mu = (y_1y_6)_2$ and $\nu = (y_2y_3y_4y_5)_2$.
  - Let $Z_j = (z_1z_2z_3z_4)_2 = S_j(\mu, \nu)$.

DES Encryption (contd)

Encryption primitive

$$e(X, J) = \mathrm{P}(S(\mathrm{E}(X) \oplus J)),$$

where X is a 32-bit message block, and J is a 48-bit round key.

- Apply 32-to-48 bit expansion: $X' = \mathrm{E}(X) = x_{32}x_1x_2 \ldots x_{32}x_1$.
- XOR with the round key: $Y = X' \oplus J$.
- Break Y in eight 6-bit parts: $Y = Y_1 \mid\mid Y_2 \mid\mid \cdots \mid\mid Y_8$.
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- Apply permutation function: $e(X, J) = \mathrm{P}(Z) = z_{16}z_7z_{20} \ldots z_4z_{25}$.

DES Encryption (contd)

E

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

P

16	7	20	21
29	12	28	17
1	15	23	26
5	18	31	10
2	8	24	14
32	27	3	9
19	13	30	6
22	11	4	25

DES Encryption: S-Boxes

S_1

14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

S_2

15	1	8	14	6	11	3	4	9	7	2	13	12	0	5	10
3	13	4	7	15	2	8	14	12	0	1	10	6	9	11	5
0	14	7	11	10	4	13	1	5	8	12	6	9	3	2	15
13	8	10	1	3	15	4	2	11	6	7	12	0	5	14	9

S_3

10	0	9	14	6	3	15	5	1	13	12	7	11	4	2	8
13	7	0	9	3	4	6	10	2	8	5	14	12	11	15	1
13	6	4	9	8	15	3	0	11	1	2	12	5	10	14	7
1	10	13	0	6	9	8	7	4	15	14	3	11	5	2	12

DES Encryption: S-Boxes (contd)

S_4

7	13	14	3	0	6	9	10	1	2	8	5	11	12	4	15
13	8	11	5	6	15	0	3	4	7	2	12	1	10	14	9
10	6	9	0	12	11	7	13	15	1	3	14	5	2	8	4
3	15	0	6	10	1	13	8	9	4	5	11	12	7	2	14

S_5

2	12	4	1	7	10	11	6	8	5	3	15	13	0	14	9
14	11	2	12	4	7	13	1	5	0	15	10	3	9	8	6
4	2	1	11	10	13	7	8	15	9	12	5	6	3	0	14
11	8	12	7	1	14	2	13	6	15	0	9	10	4	5	3

S_6

12	1	10	15	9	2	6	8	0	13	3	4	14	7	5	11
10	15	4	2	7	12	9	5	6	1	13	14	0	11	3	8
9	14	15	5	2	8	12	3	7	0	4	10	1	13	11	6
4	3	2	12	9	5	15	10	11	14	1	7	6	0	8	13

DES Encryption: S-Boxes (contd)

S_7

4	11	2	14	15	0	8	13	3	12	9	7	5	10	6	1
13	0	11	7	4	9	1	10	14	3	5	12	2	15	8	6
1	4	11	13	12	3	7	14	10	15	6	8	0	5	9	2
6	11	13	8	1	4	10	7	9	5	0	15	14	2	3	12

S_8

13	2	8	4	6	15	11	1	10	9	3	14	5	0	12	7
1	15	13	8	10	3	7	4	12	5	6	11	0	14	9	2
7	11	4	1	9	12	14	2	0	6	10	13	15	3	5	8
2	1	14	7	4	10	8	13	15	12	9	0	3	5	6	11

DES Decryption

Input: Ciphertext block $C = c_1 c_2 \ldots c_{64}$ and round keys $K_1, K_2, \ldots, K_{16}$.

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Note: DES decryption is the same as DES encryption, with the key schedule reversed.

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- AES **key schedule**: From K, generate 32-bit round keys $K_0, K_1, \ldots, K_{4l+3}$. Four round keys are used in a round.

AES (contd)

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- **State:** AES represents a 128-bit message block as a 4×4 array of octets:

$$\mu_0\mu_1\cdots\mu_{15} \equiv \begin{array}{|c|c|c|c|}\hline \mu_0 & \mu_4 & \mu_8 & \mu_{12} \\ \hline \mu_1 & \mu_5 & \mu_9 & \mu_{13} \\ \hline \mu_2 & \mu_6 & \mu_{10} & \mu_{14} \\ \hline \mu_3 & \mu_7 & \mu_{11} & \mu_{15} \\ \hline \end{array}$$

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- Each octet $A = a_7a_6\ldots a_1a_0$ in the state is identified with the element $a_7\alpha^7 + a_6\alpha^6 + \cdots + a_1\alpha + a_0$ of $\mathbb{F}_{2^8} = \mathbb{F}_2(\alpha)$, where $\alpha^8 + \alpha^4 + \alpha^3 + \alpha + 1 = 0$.

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- Each column $A_3A_2A_1A_0$ in the state is identified with the element $A_3y^3 + A_2y^2 + A_1y + A_0$ of $\mathbb{F}_{2^8}[y]$ modulo the (reducible polynomial) $y^4 + 1$.

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  $S = \text{ShiftRows}(S)$. [cyclic shift of octets in each row]
  If $i \neq l$, $S = \text{MixCols}(S)$. [operation in $\mathbb{F}_{2^8}[y] \bmod y^4 + 1$]
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- for $i = l-1, l-2, \ldots, 1, 0$ do the following:
  $S = \text{ShiftRows}^{-1}(S)$.
  $S = \text{SubState}^{-1}(S)$.
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- Let $S = (\sigma_{uv}) = \begin{array}{|c|c|c|c|}\hline \sigma_{00} & \sigma_{01} & \sigma_{02} & \sigma_{03} \\ \hline \sigma_{10} & \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \hline \sigma_{20} & \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \hline \sigma_{30} & \sigma_{31} & \sigma_{32} & \sigma_{33} \\ \hline \end{array}$ be a state of AES.

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- S maps to $\text{AddKey}(S, L_0, L_1, L_2, L_3) =$

$$\begin{array}{|c|c|c|c|}\hline \sigma_{00} \oplus \lambda_{00} & \sigma_{01} \oplus \lambda_{10} & \sigma_{02} \oplus \lambda_{20} & \sigma_{03} \oplus \lambda_{30} \\ \hline \sigma_{10} \oplus \lambda_{01} & \sigma_{11} \oplus \lambda_{11} & \sigma_{12} \oplus \lambda_{21} & \sigma_{13} \oplus \lambda_{31} \\ \hline \sigma_{20} \oplus \lambda_{02} & \sigma_{21} \oplus \lambda_{12} & \sigma_{22} \oplus \lambda_{22} & \sigma_{23} \oplus \lambda_{32} \\ \hline \sigma_{30} \oplus \lambda_{03} & \sigma_{31} \oplus \lambda_{13} & \sigma_{32} \oplus \lambda_{23} & \sigma_{33} \oplus \lambda_{33} \\ \hline \end{array}.$$

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- Let $D = d_0d_1\ldots d_6d_7 = 63 = 01100011$.
- SubOctet$(A) = C = c_0c_1\ldots c_6c_7$, where
  $c_i = b_i \oplus b_{(i+1)\mathrm{rem}8} \oplus b_{(i+2)\mathrm{rem}8} \oplus b_{(i+3)\mathrm{rem}8} \oplus b_{(i+4)\mathrm{rem}8} \oplus d_i$.

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- Let $A = a_0a_1 \ldots a_6a_7$ be an octet (an element of $\mathbb{F}_{2^8}$).
- Let $B = b_0b_1 \ldots b_6b_7 = A^{-1}$ in $\mathbb{F}_{2^8}$ (with $0^{-1} = 0$).
- Let $D = d_0d_1 \ldots d_6d_7 = 63 = 01100011$.
- SubOctet$(A) = C = c_0c_1 \ldots c_6c_7$, where
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- Let $S = \begin{array}{|c|c|c|c|}\hline \sigma_{00} & \sigma_{01} & \sigma_{02} & \sigma_{03} \\ \hline \sigma_{10} & \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \hline \sigma_{20} & \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \hline \sigma_{30} & \sigma_{31} & \sigma_{32} & \sigma_{33} \\ \hline \end{array}$ be a state.

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AES: The ShiftRows Primitive

Cyclically left rotate the r-th row by r bytes:

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- The v-th column

$$\begin{pmatrix} \sigma_{0v} \\ \sigma_{1v} \\ \sigma_{2v} \\ \sigma_{3v} \end{pmatrix} \text{ maps to } \begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \odot \begin{pmatrix} \sigma_{0v} \\ \sigma_{1v} \\ \sigma_{2v} \\ \sigma_{3v} \end{pmatrix},$$

where $\odot$ is the multiplication of $\mathbb{F}_{2^8}$.

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- ShiftRows^{-1} cyclically right rotates the r-th row by r bytes.
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    set $T = (\tau'_1 \tau'_2 \tau'_3 \tau'_0) \oplus [\alpha^{(i/t)-1} \,||\, 000000]$,
    else if ($t > 6$) and ($i \equiv 4 \pmod t$), then
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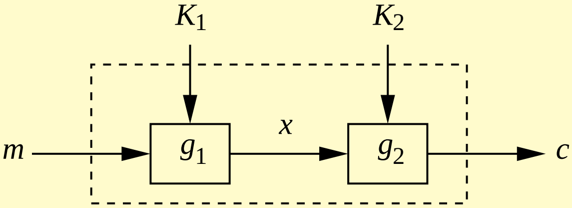
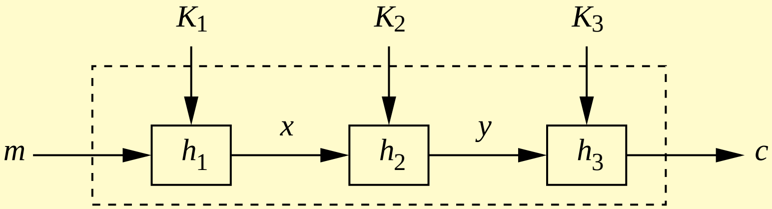
  - Generate the 32-bit key $K_i = K_{i-t} \oplus T$.

Multiple Encryption



(a) Double encryption



(b) Triple encryption

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- CFB and OFB modes act like stream ciphers

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- **Decryption:** $m_i = c_i \oplus k_i$.
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- **Vernam's one-time pad:** For a truly random key stream,

  $$\Pr(c_i = 0) = \Pr(c_i = 1) = \frac{1}{2}$$

  for each i, irrespective of the probabilities of the values assumed by m_i. This leads to **unconditional security**, that is, the knowledge of any number of plaintext-ciphertext bit pairs, does not help in decrypting a new ciphertext bit.

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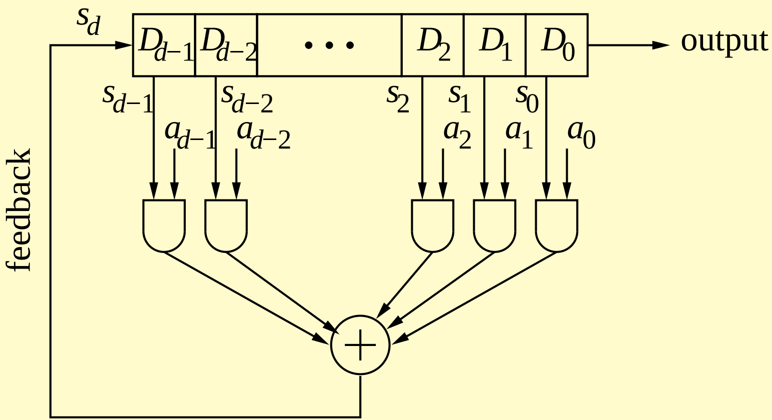
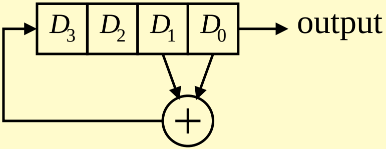
Linear Feedback Shift Register (LFSR)



LFSR: Example



LFSR: Example

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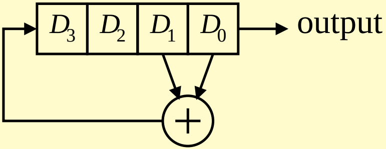

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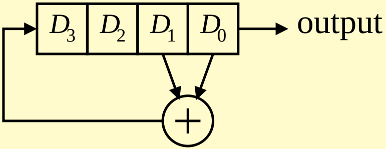

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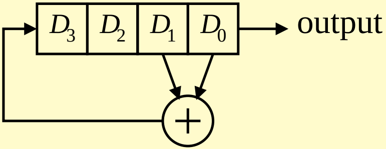

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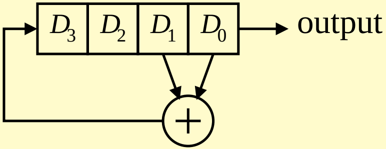

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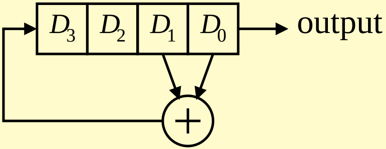

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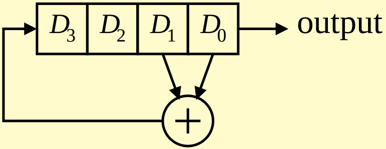

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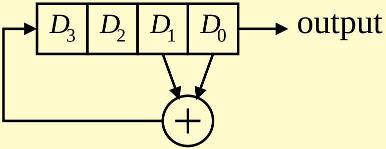

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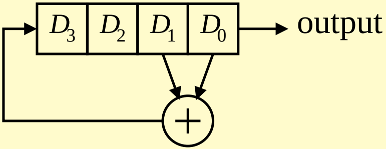

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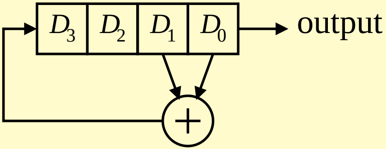

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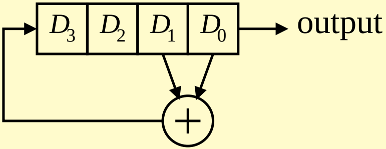

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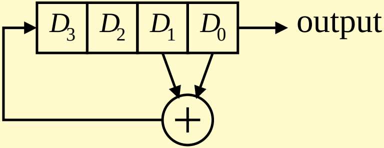

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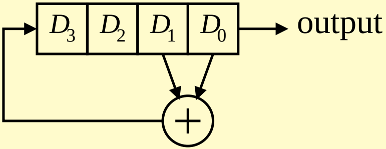

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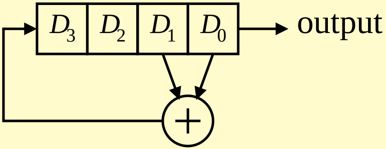

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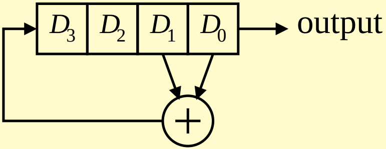

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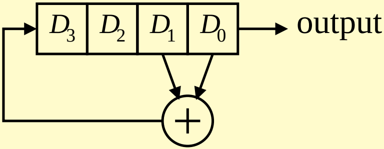

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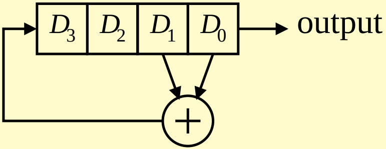

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- Each clock pulse changes the state as follows:

$$
\begin{aligned}
t_0 &= s_1 \\
t_1 &= s_2 \\
&\vdots \\
t_{d-2} &= s_{d-1} \\
t_{d-1} &\equiv a_0 s_0 + a_1 s_1 + a_2 s_2 + \cdots + a_{d-1} s_{d-1} \pmod 2.
\end{aligned}
$$

LFSR: State Transition (contd)

- In the matrix notation $\mathbf{t} \equiv \Delta_L \mathbf{s} \pmod 2$, where the **transition matrix** is

$$\Delta_L = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ a_0 & a_1 & a_2 & \cdots & a_{d-2} & a_{d-1} \end{pmatrix}.$$

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- L is a maximum-length LFSR if and only if $C_L(x)$ is a primitive polynomial of $\mathbb{F}_2[x]$.

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$$S_{i+1} \equiv \Delta_L S_i \pmod 2$$

  for $i = 1, 2, \ldots, d$. Treat each S_i as a column vector. Then,

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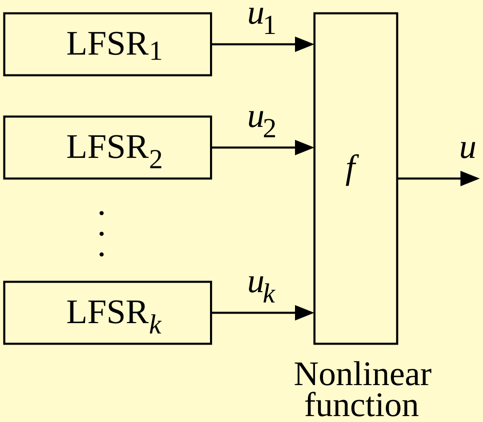
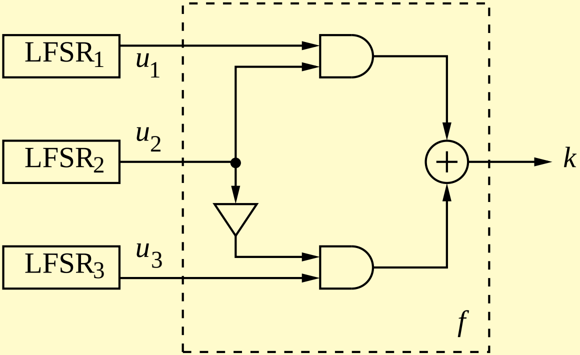
- **Remedy:** Introduce non-linearity to the LFSR output.

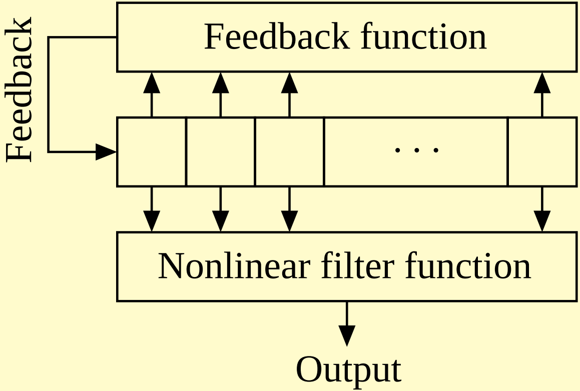
Nonlinear Combination Generator



The Geffe Generator



Nonlinear Filter Generator



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- Since hash functions map an infinite domain to finite sets, collisions must exist for any hash function.

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- **Collision resistance:** It should be difficult to find two distinct strings x, x' with $H(x) = H(x')$.

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- **Collision resistance implies second pre-image resistance.**
- **Second pre-image resistance does not imply collision resistance:** Let S be a finite set of size $\geqslant 2$ and H a cryptographic hash function. Then

$$H'(x) = \begin{cases} 0^{n+1} & \text{if } x \in S, \\ 1 \,||\, H(x) & \text{otherwise,} \end{cases}$$

is second pre-image resistant but not collision resistant.

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- **First pre-image resistance does not imply second pre-image resistance:** Let m be a product of two unknown big primes. Define $H'''(x) = (1 \,||\, x)^2 \pmod{m}$. H''' is first pre-image resistant, but not second pre-image resistant.

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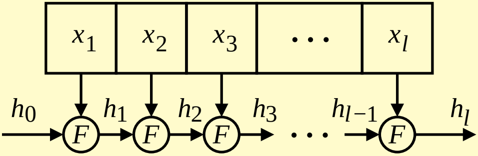

Hash Functions: Construction (contd)

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Hash Functions: Construction (contd)

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  - If F is first pre-image resistant, then H is also first pre-image resistant.

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- **Keyed hash function**
  $\text{HMAC}(M) = H(K \,||\, P \,||\, H(K \,||\, Q \,||\, M))$, where H is an unkeyed hash function, K is a key and P, Q are short padding strings.

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- Break M' into 512-bit blocks $M^{(1)}, M^{(2)}, \ldots, M^{(l)}$.
- Break each $M^{(i)} = M_0^{(i)} \mid\mid M_1^{(i)} \mid\mid \cdots \mid\mid M_{15}^{(i)}$ into sixteen 32-bit words $M_j^{(i)}$.

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- For $j = 0, 1, 2, 3, 4$, update $H_j^{(i)} := \left(t_j + H_j^{(i-1)}\right) \operatorname{rem} 2^{32}$.

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- LR^k and RR^k mean left and right rotate by k bits.

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- A collision of an n-bit hash function can be found with high probability from $O(2^{n/2})$ random hash calculations.