MTH 222 Theory of Computation

Second Mid Semester Examination (Exercise set A)

Total marks: 25 | October 2002 | Time: $1 + \epsilon$ hours

Name: **Roll Number:**

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1. Which of the following statements is/are true? (Give an explanation for each in at most two sentences.) **(2×5)**
(**Remark:** No credit will be given to a correct guess followed by an improper explanation.)

(a) If the fanout $\phi(G)$ of a CFG G is $\leqslant 2$, then $\mathcal{L}(G)$ may be infinite.

(b) $aabbaa \in \mathcal{L}(G)$, where $G := (\{a, b\}, \{S\}, S, \{S \to b \mid Sa \mid aS \mid SS\})$.

(c) The CFG G of Part (b) is ambiguous.

(d) $\mathcal{L}(G)$ is the language of the regular expression $a^*bb^*a^*$, where G is the CFG of Part (b).

(e) The union of infinitely many context-free languages may be non-context-free.

2. Let $\Sigma := \{a, b, c\}$ and $L := \{\alpha c \alpha^R c \alpha \mid \alpha \in \{a, b\}^*\}$.

(a) Show that L is not context-free. **(4)**

(b) Write L as the intersection of two context-free languages (over Σ). **(4)**

3. Let $L := \{a^{3k+1}b^{5k-2} \mid k \geqslant 1\} \subseteq \{a, b\}^*$.

(a) Write a CFG G with $\mathcal{L}(G) = L$. **(3)**

(b) Design a PDA M with $\mathcal{L}(M) = L$. **(3)**

(c) Is the PDA you designed in Part (b) a deterministic PDA? **(1)**

4. [Bonus problem] Let $\Sigma := \{a, b\}$. For $x \in \Sigma$ and $\alpha \in \Sigma^*$ define $\nu_x(\alpha) :=$ the number of occurrences of x in α. Design a PDA M with $\mathcal{L}(M) = \{\alpha \in \Sigma^* \mid \nu_b(\alpha)$ is an (integral) multiple of $\nu_a(\alpha)\}$. **(10)**

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Second Mid Semester Examination (Exercise set B)

Total marks: 25 | October 2002 | Time: $1 + \epsilon$ hours

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1. Which of the following statements is/are true? (Give an explanation for each in at most two sentences.) **(2×5)**
(**Remark:** No credit will be given to a correct guess followed by an improper explanation.)

(a) $aabbaa \in \mathcal{L}(G)$, where $G := (\{a, b\}, \{S\}, S, \{S \to \epsilon \mid Sb \mid aSa\})$.

(b) $\mathcal{L}(G)$ is the language of the regular expression $a^*b^*a^*$, where G is the CFG of Part (a).

(c) The grammar of Part (a) is ambiguous.

(d) If $\mathcal{L}(G)$ is finite for a CFG G, then the fanout $\phi(G)$ of G is $\leqslant 2$.

(e) The intersection of two context-free languages is never context-free.

2. Let $\Sigma := \{a, b, c\}$ and $L := \{\alpha a \alpha^R a \alpha \mid \alpha \in \{b, c\}^*\}$.

(a) Show that L is not context-free. **(4)**

(b) Write L as the intersection of two context-free languages (over Σ). **(4)**

3. Let $L := \{a^{5k+1}b^{3k-2} \mid k \geqslant 1\} \subseteq \{a, b\}^*$.

(a) Write a CFG G with $\mathcal{L}(G) = L$. **(3)**

(b) Design a PDA M with $\mathcal{L}(M) = L$. **(3)**

(c) Is the PDA you designed in Part (b) a deterministic PDA? **(1)**

4. [Bonus problem] Let $\Sigma := \{a, b\}$. For $x \in \Sigma$ and $\alpha \in \Sigma^*$ define $\nu_x(\alpha) :=$ the number of occurrences of x in α. Design a PDA M with $\mathcal{L}(M) = \{\alpha \in \Sigma^* \mid \nu_a(\alpha)$ is an (integral) multiple of $\nu_b(\alpha)\}$. **(10)**