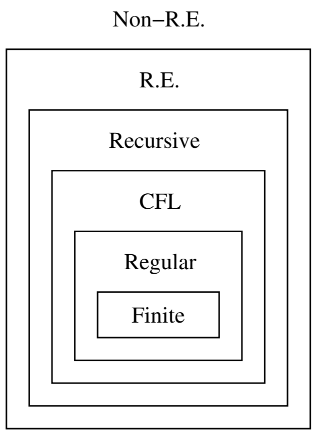
MTH 222 Theory of Computation

End Semester Examination

Total marks: 50 | December 4, 2002 | Time: $3 + \epsilon$ hours

---

1. Consider the hierarchy of languages shown in the adjacent figure. Place each of the following languages at the proper place in the hierarchy, that is, deduce whether for each $i = 1, \ldots, 4$ the language L_i as defined below is 'finite' or 'regular-and-infinite' or 'context-free-but-not-regular' or 'recursive-but-non-context-free' or 'R.E.-but-not-recursive' or 'non-R.E.'. You should provide sufficient justifications for your judgments. **(4×4)**



(a) The set L_1 of all strings $\alpha \in \{a, b, c\}^*$ containing an equal number of occurrences of a's, b's and c's.

(b) The complement of L_1, that is, $L_2 := \{a, b, c\}^* \setminus L_1$.

(c) $L_3 := \mathcal{L}(G)$, where G is the grammar

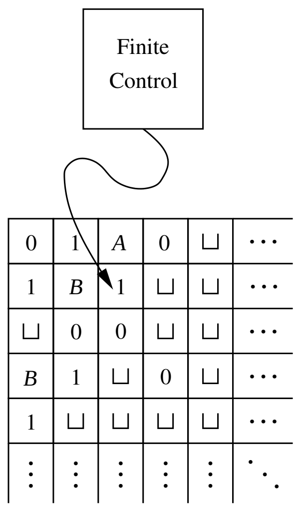
$$\Big(\{a, b\}, \{S, T\}, S, \{S \to \epsilon \mid abTS, T \to \epsilon \mid Tb\}\Big).$$

(d) $L_4 := \Big\{\langle M_i \rangle \mid \epsilon \notin \mathcal{L}(M_i)\Big\}$, where $\langle M_i \rangle$ is the code for the i-th Turing machine with the binary input alphabet.

2. Let M be a (one-tape one-head) Turing machine with every transition of the form $\delta(q, X) = (p, Y, R)$, that is, M never makes a left move of its head. Show that $\mathcal{L}(M)$ is regular. (**Remark:** So the inability of the head to move left is a serious loss of generality!) **(8)**

3. Consider a Turing Machine M with a two-dimensional tape as shown in the adjacent figure. Initially the input string is provided horizontally at the topmost row starting from the first column, the finite control of M is in the start state, and the head is positioned at the top-left cell of the tape. Each move of M is dependent on the state of the finite control and on the tape symbol scanned by the head. During each move the finite control switches to a new state (which may be the same as the old state), the tape symbol currently scanned by the head is overwritten by a new symbol (which may be the same as the old symbol), and the head moves by one cell in one of the four directions north, east, west and south. M accepts by entering a final state. **(10)**

Show that $\mathcal{L}(M)$ is R.E.



4. For a string $\alpha \in \{0, 1\}^+$ define $\mathrm{val}(\alpha)$ to be the non-negative integer with a binary representation α. (In this exercise leading zeros are allowed in binary representations.) Define the language

$$L_{gt} := \Big\{\alpha\#\beta \mid \alpha, \beta \in \{0, 1\}^+ \text{ and } \mathrm{val}(\alpha) > \mathrm{val}(\beta)\Big\} \subseteq \{0, 1, \#\}^*.$$

(a) Design an unrestricted grammar G with $\mathcal{L}(G) = L_{gt}$. You must elaborate how your grammar works, that is, how it accepts the members of L_{gt} and rejects other strings. **(8)**

(b) Design a Turing machine M with $\mathcal{L}(M) = L_{gt}$. You need not specify the official details (like the transition diagram) of your machine, but you must clearly explain how your machine works. You may use multiple tapes, if necessary. **(6)**

(c) Is L_{gt} recursive? **(2)**