MTH 215 Number Theory : Quiz 2A

Total marks: 15 | April 11, 2002 | Time: 30 minutes

Name: **Roll Number:**

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1. Let $\xi := \pi^2 = 9.8696044010\ldots$. Find three rational numbers a/b, $b > 0$, such that $\left|\xi - \frac{a}{b}\right| < \frac{1}{b^2}$. **(3)**

2. Let ξ and ξ' be (arbitrary) irrational numbers such that $\xi + \xi'$ is also irrational. Let r_n and r'_n be the nth convergents to ξ and ξ' respectively. Prove or disprove: The nth convergent to $\xi + \xi'$ is $r_n + r'_n$. **(3)**

3. Which of the following Diophantine equations is/are solvable in positive integer values of x, y, z and w ?

(a) $x^8 + 16y^{12} = zw^6$. **(2)**

(b) $x^8 + 16y^{12} = z^2w^6$. **(2)**

4. Do there exist *Pythagorean quadruples*, that is, $(a, b, c, d) \in \mathbb{N}^4$ satisfying $a^2 + b^2 + c^2 = d^2$? If yes, how many with $\gcd(a, b, c) = 1$? If not, why? **(5)**

MTH 215 Number Theory : Quiz 2B

Total marks: 15 | April 11, 2002 | Time: 30 minutes

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1. Let $\xi := 2\pi = 6.2831853071\ldots$. Find three rational numbers a/b, $b > 0$, such that $\left|\xi - \dfrac{a}{b}\right| < \dfrac{1}{b^2}$. **(3)**

2. Let ξ and ξ' be (arbitrary) irrational numbers such that $\xi\xi'$ is also irrational. Let r_n and r'_n be the nth convergents to ξ and ξ' respectively. Prove or disprove: The nth convergent to $\xi\xi'$ is $r_n r'_n$. **(3)**

3. Which of the following Diophantine equations is/are solvable in positive integer values of x, y, z and w ?

(a) $16x^8 + y^{12} = z^6 w$. **(2)**

(b) $16x^8 + y^{12} = z^6 w^2$. **(2)**

4. Do there exist *Pythagorean quadruples*, that is, $(a, b, c, d) \in \mathbb{N}^4$ satisfying $a^2 + b^2 + c^2 = d^2$? If yes, how many with $\gcd(a, b, c) = 1$? If not, why? **(5)**