MTH 215 Number Theory : Quiz 1

Total marks: 15 February 28, 2002 Time: 30 minutes

Name: **Roll Number:**

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1. **(a)** List *all* two-digit integers that can be formed by picking up without repetition (decimal) digits from your roll number. **(2)**

(**Remark:** 09 is not a two-digit number. Y is not a decimal digit. If your roll number contains repeating digits, say, two or more occurrences of 9, then 99 should be in your list. If 9 occurs only once (or it does not), then 99 should not be in your list. For example, the list for the roll number Y1021 should be $10, 11, 12, 20, 21$.)

(b) Let $m_1, \ldots, m_k$ be the integers constructed in Part (a). For which of the values of $j \in \{1, \ldots, k\}$ is the group $\mathbb{Z}_{m_j}^*$ cyclic? **(2)**

(c) If no $\mathbb{Z}_{m_j}^*$ of Part (b) is cyclic, take $m := 101$, otherwise take m to be the largest of m_j, $1 \leqslant j \leqslant k$, such that $\mathbb{Z}_{m_j}^*$ is cyclic. Find a primitive root modulo m. **(3)**

2. Let $a \in \mathbb{N}$ be your roll number with Y (if present) replaced by 9. Compute the Legendre symbol $\left(\frac{a}{100\,003}\right)$. **(3)**
(**Remark:** 100003 is a prime.)

3. Show that for any $h \in \mathbb{N}$ there exist $m, a \in \mathbb{N}$ with $\gcd(a, m) = 1$ such that $\operatorname{ord}_m a = h$. (**Hint:** Consider the prime factorization of h.) **(5)**