Searching Elements in an Array: Linear and Binary Search

Searching

- Check if a given element (called ***key***) occurs in the array.
  - Example: array of student records; rollno can be the key.
- Two methods to be discussed:
  - If the array elements are unsorted.
    - Linear search
  - If the array elements are sorted.
    - Binary search

Linear Search

Basic Concept

- **Basic idea:**
  - **Start at the beginning of the array.**
  - **Inspect elements one by one to see if it matches the key.**
- **Time complexity:**
  - **A measure of how long an algorithm takes to run.**
  - **If there are *n* elements in the array:**
    - **Best case:**

      match found in first element (1 search operation)
    - **Worst case:**

      no match found, or match found in the last element (n search operations)
    - **Average case:**

      (n + 1) / 2 search operations

Contd.

```
/* The function returns the array index where the
   match is found. It returns -1 if there is no
   match.    */

int  linear_search (int a[], int size, int key)
{
   int pos = 0;
   while ((pos < size) && (a[pos] != key))
      pos++;
   if (pos < size)
      return pos;  /* Return the position of match */
   return -1;      /* No match found */
}
```

Contd.

```
int x[]= {12, -3, 78, 67, 6, 50, 19, 10};
```

- **Trace the following calls :**

  search (x, 8, 6) ; ← **Returns 4**

  search (x, 8, 5) ; ← **Returns -1**

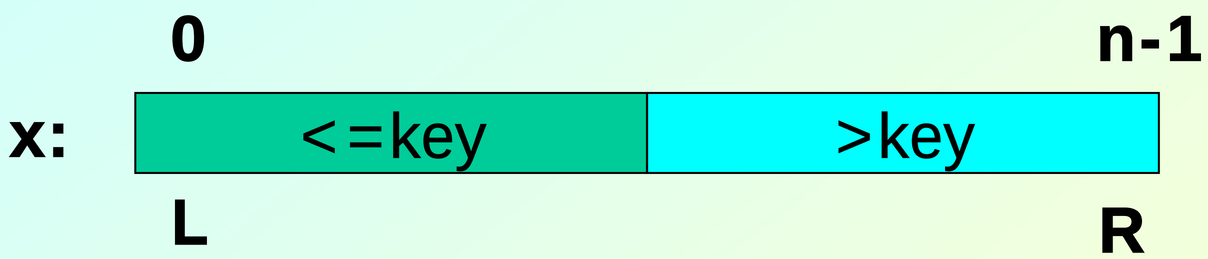
Binary Search

Basic Concept

- Binary search works if the array is ***sorted***.
  - Look for the target in the middle.
  - If you don't find it, you can ignore half of the array, and repeat the process with the other half.
- In every step, we reduce the number of elements to search in by half.

The Basic Strategy

- **What we want?**
  - **Find split between values larger and smaller than key:**



  - **Situation while searching:**
    - **Initially L and R contains the indices of first and last elements.**
  - **Look at the element at index [(L+R)/2].**
    - **Move L or R to the middle depending on the outcome of test.**

Contd.

```
/* If key appears in x[0..size-1], return its location, pos
   such that x[pos]==key. If not found, return -1 */

int bin_search (int x[], int size, int key)
{
  int L, R, mid;
  _________________;
  while ( ____________ )
  {
       __________________;
  }
  _________________ ;
}
```

The basic search iteration

```
int bin_search (int x[], int size, int key)
{
  int L, R, mid;
  _________________;
  while ( ____________ )
  {
     mid = (L + R) / 2;
     if (x[mid] <= key)
        L = mid;
     else R = mid;
  }
  _________________ ;
}
```

Loop termination

```
int bin_search (int x[], int size, int key)
{
  int L, R, mid;
  _________________;
  while ( L+1 != R )
  {
     mid = (L + R) / 2;
     if (x[mid] <= key)
        L = mid;
     else R = mid;
  }
  _________________ ;
}
```

Return result

```
int bin_search (int x[], int size, int key)
{
  int L, R, mid;
  _________________;
  while ( L+1 != R )
  {
     mid = (L + R) / 2;
     if (x[mid] <= key)
         L = mid;
     else R = mid;
  }
  if (L >= 0 && x[L] == key)  return L;
  else return -1;
}
```

Initialization

```
int bin_search (int x[], int size, int key)
{
  int L, R, mid;
  L = -1;   R = size;
  while ( L+1 != R )
  {
     mid = (L + R) / 2;
     if (x[mid] <= key)
        L = mid;
     else R = mid;
  }
  if (L >= 0 && x[L] == key) return L;
  else return -1;
}
```

Binary Search Examples

Sorted array

-17	-5	3	6	12	21	45	63	50

Trace :

binsearch (x, 9, 3); →

binsearch (x, 9, 145);

binsearch (x, 9, 45);

L= –1; R= 9; x[4]=12;
L= –1; R=4; x[1]= –5;
L= 1; R=4; x[2]=3;
L=2; R=4; x[3]=6;
L=2; R=3; return L;

We may modify the algorithm by checking equality with x[mid].

Is it worth the trouble ?

- Suppose that the array **x** has 1000 elements.
- Ordinary search
- If ***key*** is a member of **x**, it would require 500 comparisons on the average.
- Binary search
  - after 1st compare, left with 500 elements.
  - after 2nd compare, left with 250 elements.
  - After at most 10 steps, you are done.

Time Complexity

- If there are n elements in the array.
  - Number of searches required:

    $\log_2 n$

- For n = 64 (say).
  - Initially, list size = 64.
  - After first compare, list size = 32.
  - After second compare, list size = 16.
  - After third compare, list size = 8.
  - …….
  - After sixth compare, list size = 1.

$2^k = n$, where k is the number of steps.

$\log_2 64 = 6$

$\log_2 1024 = 10$

Sorting

The Basic Problem

- Given an array

  x[0], x[1], ... , x[size-1]

  reorder entries so that

  x[0] <= x[1] <= . . . <= x[size-1]

  - List is in non-decreasing order.

- We can also sort a list of elements in non-increasing order.

Example

- **Original list:**

  10, 30, 20, 80, 70, 10, 60, 40, 70

- **Sorted in non-decreasing order:**

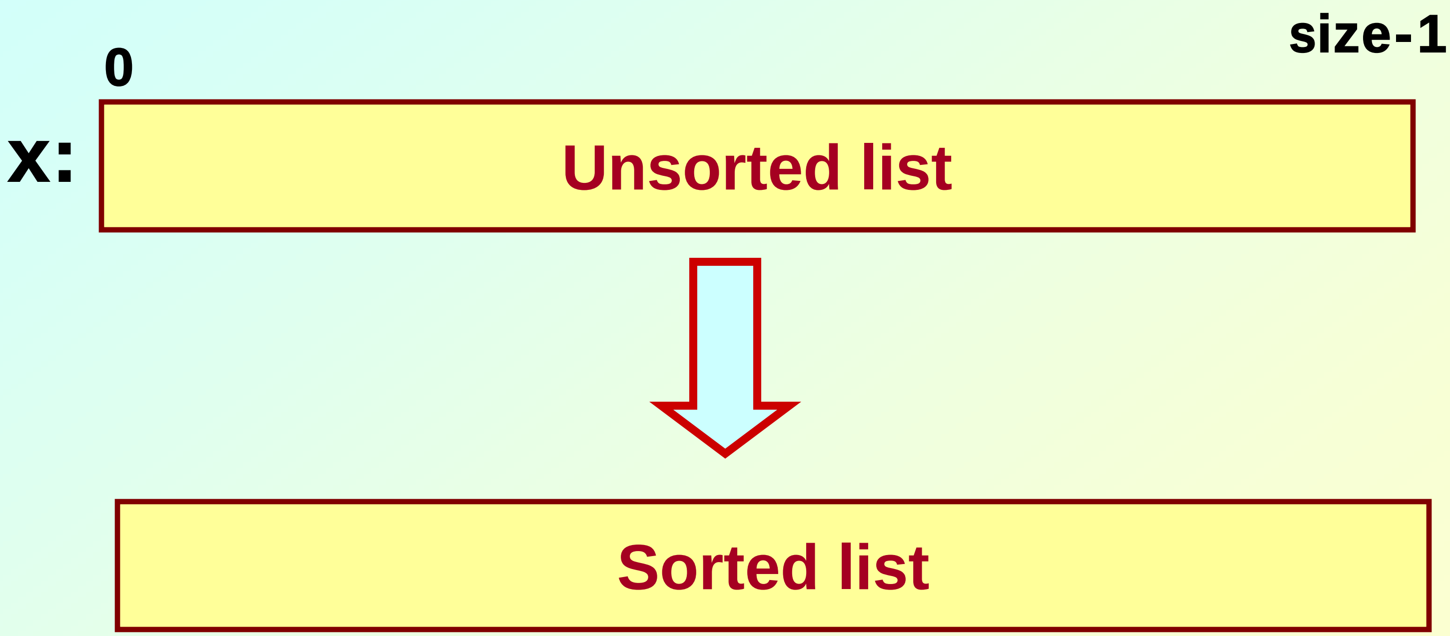
  10, 10, 20, 30, 40, 60, 70, 70, 80

- **Sorted in non-increasing order:**

  80, 70, 70, 60, 40, 30, 20, 10, 10

Sorting Problem

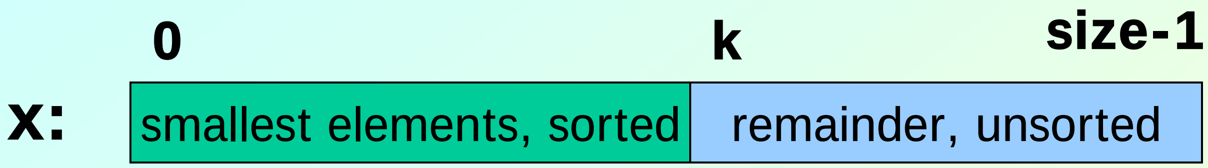
- **What we want ?**
  - Data sorted in order



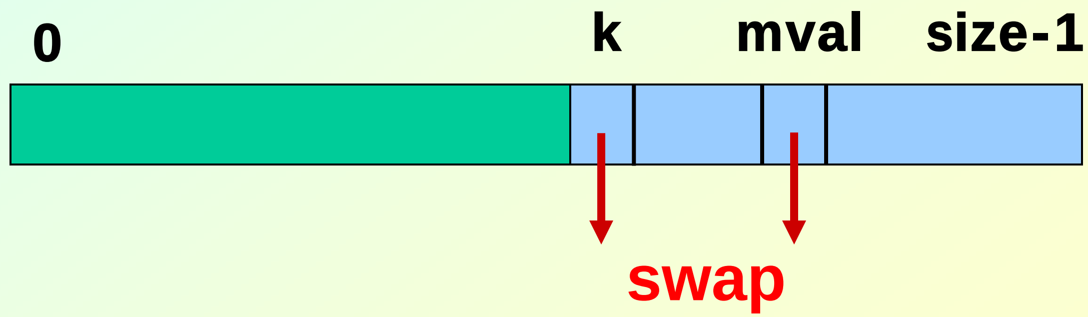
Selection Sort

How it works?

- **General situation :**



- **Step :**
  - **Find smallest element, mval, in x[k..size-1]**
  - **Swap smallest element with x[k], then increase k.**



Subproblem

```
/* Yield index of smallest element in x[k..size-1];*/

int min_loc (int x[], int k, int size)
{
    int j, pos;

    pos = k;
    for (j=k+1; j<size; j++)
        if (x[j] < x[pos])
            pos = j;
    return pos;
}
```

The main sorting function

```
/* Sort x[0..size-1] in non-decreasing order */

int sel_sort (int x[], int size)
{  int k, m;

   for (k=0; k<size-1; k++)
   {
      m = min_loc (x, k, size);
      temp = a[k];
      a[k] = a[m];
      a[m] = temp;
   }
}
```

SWAP

```
int main()
{
  int x[ ]={-45,89,-65,87,0,3,-23,19,56,21,76,-50};
  int i;
  for(i=0;i<12;i++)
      printf("%d ",x[i]);
  printf("\n");
  sel_sort(x,12);
  for(i=0;i<12;i++)
      printf("%d ",x[i]);
  printf("\n");
}
```

```
-45 89 -65 87 0 3 -23 19 56 21 76 -50
-65 -50 -45 -23 0 3 19 21 56 76 87 89
```

Example

x:	3	12	-5	6	142	21	-17	45

x:	-17	12	-5	6	142	21	3	45

x:	-17	-5	12	6	142	21	3	45

x:	-17	-5	3	6	142	21	12	45

x:	-17	-5	3	6	142	21	12	45

x:	-17	-5	3	6	12	21	142	45

x:	-17	-5	3	6	12	21	142	45

x:	-17	-5	3	6	12	21	45	142

Analysis

- How many steps are needed to sort *n* items ?
  - Total number of steps *proportional* to n^2.
  - No. of comparisons?

$$(n-1)+(n-2)+\ldots\ldots+1 = n(n-1)/2$$

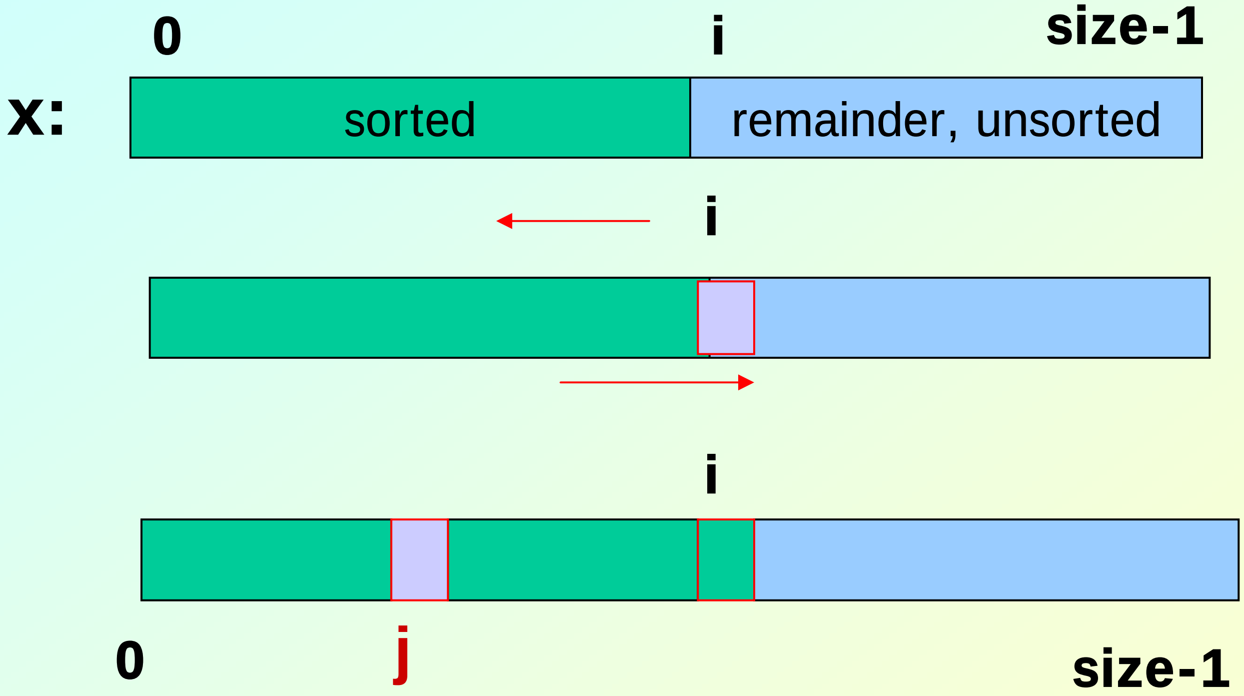
Of the order of n^2

  - Worst Case? Best Case? Average Case?

Insertion Sort

How it works?

- **General situation :**



Compare and shift till x[i] is larger.

Insertion Sort

```
void insert_sort (int list[], int size)
{
  int i,j,item;

   for (i=1; i<size; i++)
      {
       item = list[i] ;
       for (j=i-1; (j>=0)&& (list[j] > i); j--)
             list[j+1] = list[j];
       list[j+1] = item ;
   }
}
```

```c
int main()
{
  int x[ ]={-45,89,-65,87,0,3,-23,19,56,21,76,-50};
  int i;
  for(i=0;i<12;i++)
      printf("%d ",x[i]);
  printf("\n");
  insert_sort(x,12);
  for(i=0;i<12;i++)
      printf("%d ",x[i]);
  printf("\n");
}
```

```
-45 89 -65 87 0 3 -23 19 56 21 76 -50
-65 -50 -45 -23 0 3 19 21 56 76 87 89
```

Time Complexity

- **Number of comparisons and shifting:**
  - **Worst case?**

    $1 + 2 + 3 + \ldots\ldots + (n-1) = n(n-1)/2$

  - **Best case?**

    $1 + 1 + \ldots\ldots$ to $(n-1)$ terms $= (n-1)$

Bubble Sort

How it works?

- **The sorting process proceeds in several passes.**
  - **In every pass we go on comparing neighboring pairs, and swap them if out of order.**
  - **In every pass, the largest of the elements under considering will *bubble* to the top (i.e., the right).**

```
10   5  17  11  -3  12
5   10  17  11  -3  12
5   10  17  11  -3  12
5   10  11  17  -3  12
5   10  11  -3  17  12
5   10  11  -3  12  17
```

Largest (→ 17)

- **How the passes proceed?**
  - In pass 1, we consider index 0 to n-1.
  - In pass 2, we consider index 0 to n-2.
  - In pass 3, we consider index 0 to n-3.
  - ……
  - ……
  - In pass n-1, we consider index 0 to 1.

Bubble Sort

```
void swap(int *x, int *y)
{
  int tmp = *x;
  *x = *y;
  *y = tmp;
}
```

```
void bubble_sort
         (int x[], int n)
{
  int i,j;

  for (i=n-1; i>0; i--)
    for (j=0; j<i; j++)
      if (x[j] > x[j+1])
        swap(&x[j],&x[j+1]);
}
```

```
int main()
{
  int x[ ]={-45,89,-65,87,0,3,-23,19,56,21,76,-50};
  int i;
  for(i=0;i<12;i++)
      printf("%d ",x[i]);
  printf("\n");
  bubble_sort(x,12);
  for(i=0;i<12;i++)
      printf("%d ",x[i]);
  printf("\n");
}
```

```
-45 89 -65 87 0 3 -23 19 56 21 76 -50
-65 -50 -45 -23 0 3 19 21 56 76 87 89
```

Time Complexity

- **Number of comparisons :**
  - **Worst case?**

    $$1 + 2 + 3 + \ldots\ldots + (n-1) = n(n-1)/2$$

  - **Best case?**

    Same

- **How do you make best case with (n-1) comparisons only?**
  - **By maintaining a variable `flag`, to check if there has been any swaps in a given pass.**
  - **If not, the array is already sorted.**

```
void bubble_sort
         (int x[], int n)
{
  int i,j;
  int flag = 0;

  for (i=n-1; i>0; i--)
  {
      for (j=0; j<i; j++)
      if (x[j] > x[j+1])
      {
         swap(&x[j],&x[j+1]);
         flag = 1;
      }
      if (flag == 0) return;
  }
```

Some Efficient Sorting Algorithms

- **Two of the most popular sorting algorithms are based on divide-and-conquer approach.**
  - **Quick sort**
  - **Merge sort**
- **Basic concept (divide-and-conquer method):**

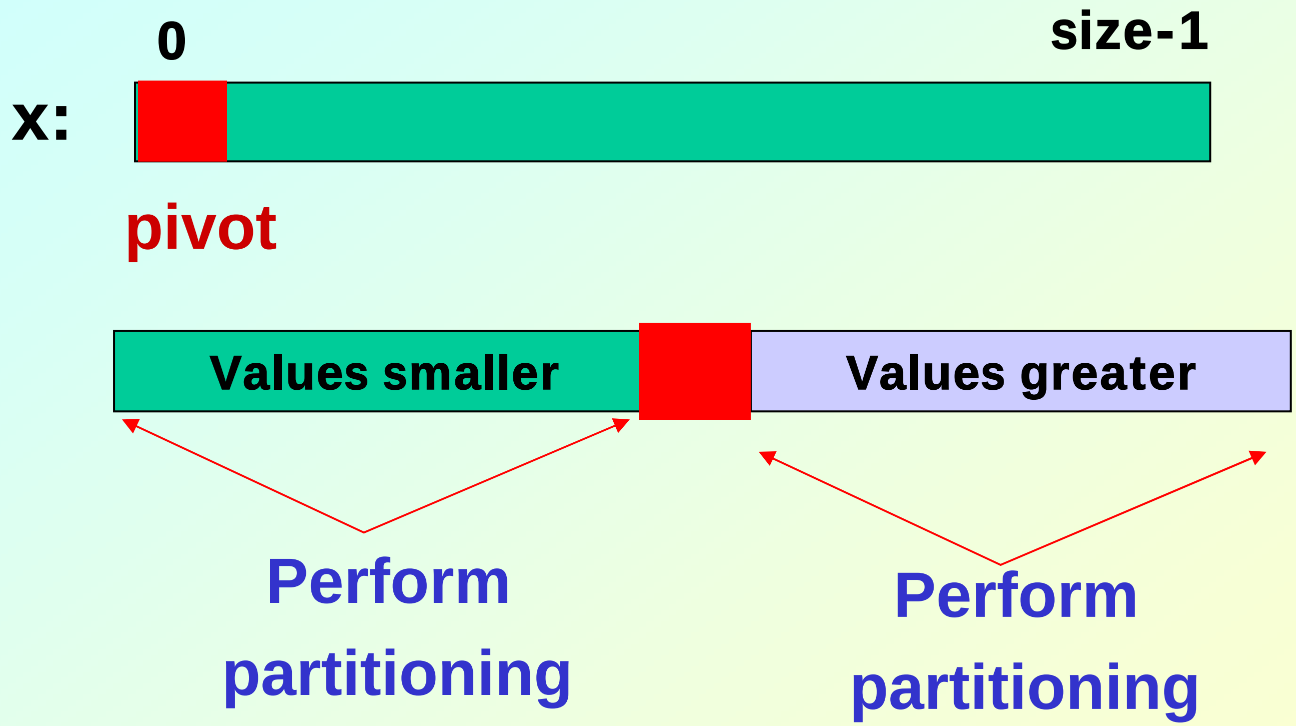
```
sort (list)
{
   if the list has length greater than 1
   {
      Partition the list into lowlist and highlist;
      sort (lowlist);
      sort (highlist);
      combine (lowlist, highlist);
   }
}
```

Quick Sort

How it works?

- **At every step, we select a *pivot element* in the list (usually the first element).**
  - **We put the pivot element in the *final position* of the sorted list.**
  - **All the elements less than or equal to the pivot element are to the left.**
  - **All the elements greater than the pivot element are to the right.**

Partitioning



Example

26	33	35	29	19	12	22
	22	35	29	19	12	33
	22	12	29	19	35	33
	22	12	19	29	35	33
19	22	12	26	29	35	33

The partitioning process

Recursively carry out the partitioning

```
void print (int x[], int low, int high)
{
   int i;
   for(i=low; i<=high; i++)
      printf(" %d", x[i]);
   printf("\n");
}
```

```
void swap (int *a, int *b)
{
  int tmp=*a;
  *a=*b;
  *b=tmp;
}
```

```
void partition (int x[], int low, int high)
{
   int i = low+1,  j = high;
   int pivot = x[low];
   if (low >= high) return;
   while (i<j)  {
      while ((x[i]<pivot) && (i<high))  i++;
      while ((x[j]>=pivot) && (j>low))  j--;
      if (i<j)  swap (&x[i], &x[j]);
   }
   if (j==high)  {
      swap (&x[j], &x[low]);
      partition (x, low, high-1);
   }
   else
      if (i==low+1)
         partition (x, low+1, high);
      else {
             swap (&x[j], &x[low]);
             partition (x, low, j-1);
             partition (x, j+1, high);
           }
}
```

```
int main (int argc, char *argv[])
{
   int *x;
   int i=0;
   int num;

   num = argc - 1;
   x = (int *) malloc(num * sizeof(int));

   for (i=0; i<num; i++)
     x[i] = atoi(argv[i+1]);

   partition(x,0,num-1);

   printf(“Sorted list: ");
   print (x,0,num-1);
}
```

Trace of Partitioning

```
./a.out 45 -56 78 90 -3 -6 123 0 -3 45 69 68
```

```
45 -56 78 90 -3 -6 123 0 -3 45 69 68

-6 -56 -3   0  -3  45  123 90 78 45 69 68

-56  -6 -3   0  -3       68 90 78 45 69  123

           -3   0 -3       45 68 78 90 69

                -3  0              69    78    90
```

```
Sorted list:  -56 -6 -3 -3 0 45 45 68 69 78 90 123
```

Time Complexity

- **Worst case:**

  n^2 ==> list is already sorted

- **Average case:**

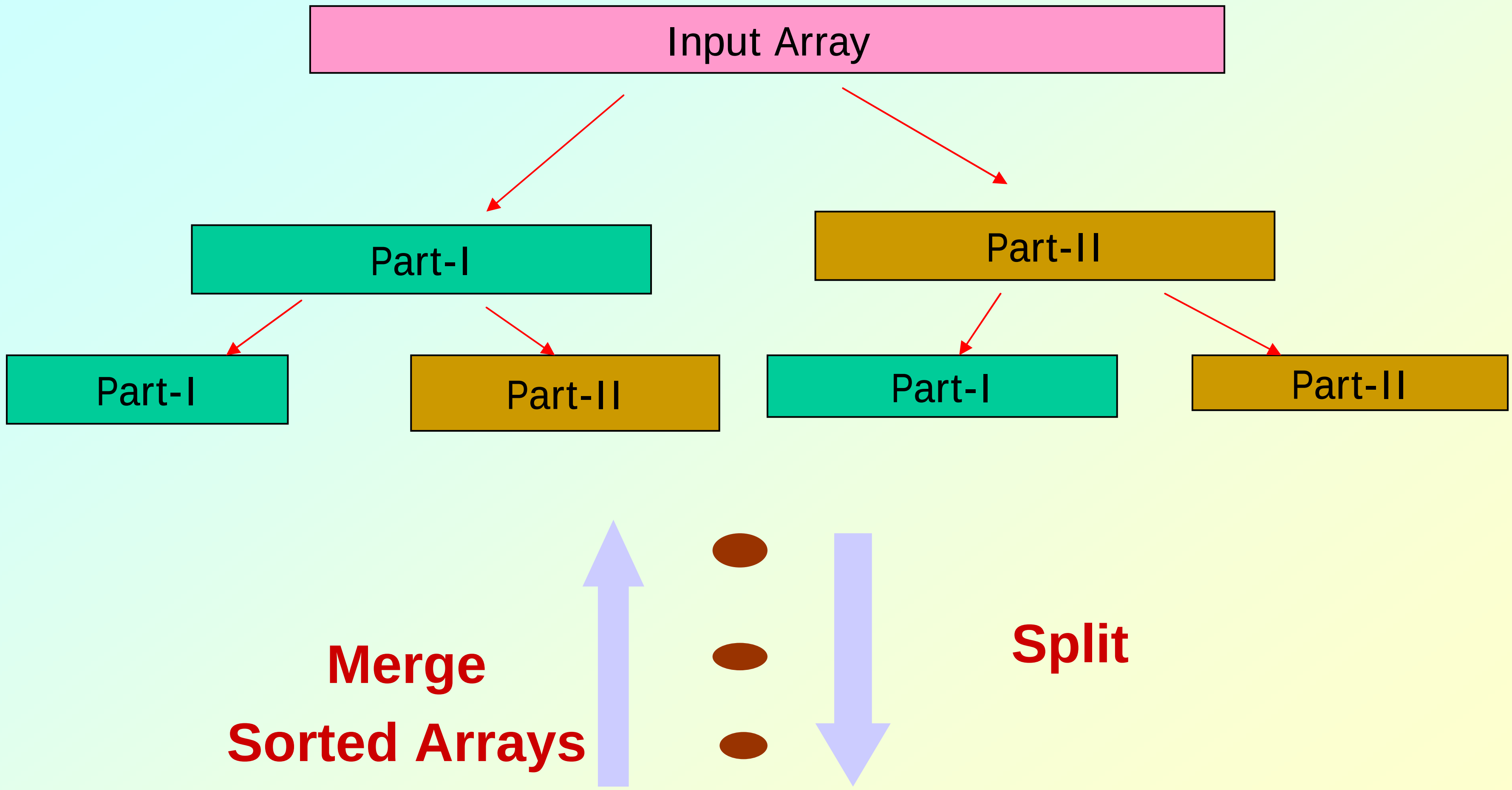
  $n \log_2 n$

- **Statistically, quick sort has been found to be one of the fastest algorithms.**

- **Corollary of quick sort:**
  - **Given a list of numbers stored in an array, determine how many numbers are smaller than a given number p?**
  - **Given a list of integers (negative and non-negative), reorder the list so that all negative numbers precede the non-negative ones.**

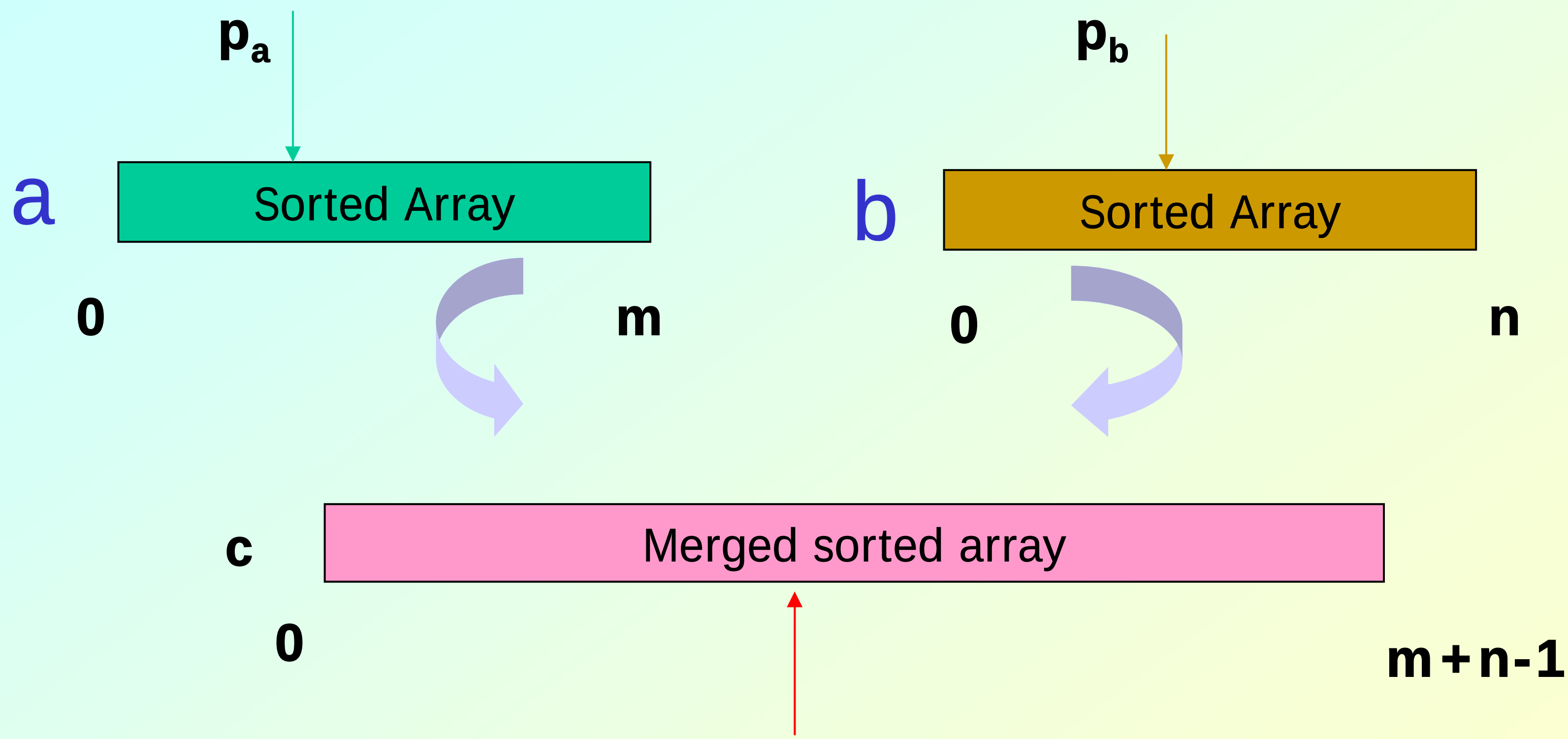
Merge Sort

Merge Sort



Input Array
Part-I
Part-II
Part-I
Part-II
Part-I
Part-II
Merge
Sorted Arrays
Split

Merging two sorted arrays



Move and copy elements pointed by p_a if its value is smaller than the element pointed by p_b in (m+n-1) operations; otherwise, copy elements pointed by p_b .

Example

- Initial array A contains 14 elements:
  - 66, 33, 40, 22, 55, 88, 60, 11, 80, 20, 50, 44, 77, 30
- Pass 1 :: Merge each pair of elements
  - (33, 66) (22, 40) (55, 88) (11, 60) (20, 80) (44, 50) (30, 70)
- Pass 2 :: Merge each pair of pairs
  - (22, 33, 40, 66) (11, 55, 60, 88) (20, 44, 50, 80) (30, 77)
- Pass 3 :: Merge each pair of sorted quadruplets
  - (11, 22, 33, 40, 55, 60, 66, 88) (20, 30, 44, 50, 77, 80)
- Pass 4 :: Merge the two sorted subarrays to get the final list
  - (11, 20, 22, 30, 33, 40, 44, 50, 55, 60, 66, 77, 80, 88)

```
void merge_sort (int *a, int n)
{
   int i, j, k, m;
   int *b, *c;

   if (n>1)  {
      k = n/2;   m = n-k;
      b = (int *) calloc(k,sizeof(int));
      c = (int *) calloc(m,sizeof(int));
      for (i=0; i<k; i++)
         b[i]=a[i];
      for (j=k; j<n; j++)
         c[j-l]=a[j];

      merge_sort (b, k);
      merge_sort (c, m);
      merge (b, c, a, k, m);
      free(b); free(c);
   }
}
```

```c
void merge (int *a, int *b, int *c, int m, int n)
{
   int i, j, k, p;

   i = j = k = 0;

   do  {
     if (a[i] < b[j])  {
       c[k]=a[i]; i=i+1;
     }
     else  {
       c[k]=b[j]; j=j+1;
     }
     k++;
   }  while ((i<m) && (j<n));

   if (i == m) {
      for (p=j; p<n; p++)  { c[k]=b[p]; k++; }
   }
   else  {
      for (p=i; p<m; p++)  { c[k]=a[p]; k++; }
   }
}
```

```
main()
{
    int i, num;
    int a[ ] = {-56,23,43,-5,-3,0,123,-35,87,56,75,80};

    for (i=0;i<12;i++)
       printf ("%d ",a[i]);
    printf ("\n");

    merge_sort (a, 12);

    printf (“\nSorted list:");
    for (i=0;i<12;i++)
       printf (“ %d", a[i]);
    printf ("\n");
}
```