

Linear algebraic and polynomial methods and applications to combinatorial problems

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Linear algebraic arguments and polynomials have found huge attention in recent times especially due to their applications to many seemingly difficult combinatorial problems. For instance, the *finite field Kakeya conjecture* proved by Dvir [2] in 2008, *Joints problem* solved by Katz and Guth [3,5,6] in 2010, *Erdős distinct distance conjecture* proved by Katz and Guth [7] in 2015, and *Cap-set conjecture* proved by Ellenberg and Gijswijt [8] in 2017 are all applications of polynomial methods. In all of the above proofs, one needs to convert the combinatorial problem into a problem on points in some vector space; find a non-zero polynomial that vanishes on these points; and use this polynomial to attack the problem. In linear algebraic proofs, one may use the linear independence of vectors in some vector space to obtain various results (see Babai and Frankl [1], Jukna [4]). We have also obtained results for a few combinatorial problems using Linear algebraic methods and polynomial methods (see [9,10,11]). In this talk, we introduce the linear algebraic methods and polynomial methods and explore few applications of these methods.

References

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