Distance Constrained Graph Labeling

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In a cellular network, the channel assignment problem (CAP) is the task of assigning channels (frequency, time, code etc.) to the cells (base stations) satisfying the constraints to avoid channel interference. The degree of interference between base stations is related to the distance between them; closer the base stations stronger the interference. So in order to avoid interference, closer base stations should have higher channel difference. The aim is to allocate channels such that span (bandwidth) is minimized. The span is the difference between the least and the highest channels used. A channel assignment problem can be modeled on a graph, whose vertices represent the cells and edges represent possible communications and hence, interferences. Usually, most of these graph theoretic models of a CAP assume communication in both directions (duplex) between two transmitters/receivers and hence the CAP is modeled on a simple graph. A channel assignment problem can also be modeled on an oriented graph under half-duplex setting in which at most one way transmission is effective between any two adjacent transmitters/receivers in a network.

Here, we will discuss some results to find optimal bounds of the span of different channel assignment problems modeled on undirected graphs.

We denote the set of vertices and edges of a simple graph G by $V(G)$ and $E(G)$, respectively. The *distance* $d(u,v)$ between two vertices u and v of G is the *length* (number of edges) of a shortest path connecting u and v. The *diameter* $diam(G)$ of G is the maximum $d(u,v)$ taken over every pair of vertices u, v of G.

An n-$L(\delta_1, \delta_2, \cdots, \delta_k)$-*labeling* of a simple graph G is a mapping $f : V(G) \to \{0, 1, \ldots, n\}$ such that $|f(u) - f(v)| \geq \delta_i$ when u and v are connected by a path of length i for all $\delta_i \in \mathbb{Z}_{\geq 0}$ and $i \in \{1, 2, \ldots, k\}$. Here $k + 1$ is the *reuse distance* and δ_i's are the *frequency separators*. The $L(\delta_1, \delta_2, \cdots, \delta_k)$ *labeling span* $\lambda_{\delta_1,\delta_2,\cdots,\delta_k}(G)$ of a graph G is the minimum n such that G admits an n-$L(\delta_1, \delta_2, \cdots, \delta_k)$-labeling. For a family $\mathcal{F}$ of graphs, $\lambda_{\delta_1,\delta_2,\cdots,\delta_k}(\mathcal{F}) = \max\{\lambda_{\delta_1,\delta_2,\cdots,\delta_k}(G) | G \in \mathcal{F}\}$.

The $L(k, k-1, \cdots, 1)$-labeling is popularly known as the *radio k-coloring* [2] whereas the corresponding span is known as the *radio k-chromatic number* [2], denoted by $rc_k(G)$.

We will discuss a general technique for computing the lower bound for $rc_k(G)$ of a general graph G and derive a formula for it. This answers the Question asked by Panigrahi [7]. Using this formula we compute lower bounds of $rc_k(\cdot)$ for several graphs and provide alternative short proofs for a number of previously established tight lower bounds [4]. In particular, we progress on Conjecture by Kchikech et al. [5] by improving the previously known lower bound by a linear additive factor of k.

We will also discuss the $L(3, 2, 1)$-labeling of the infinite triangular grid $\mathcal{L}_6$. We settled the Conjecture proposed by Calamoneri [1] by improving the lower bound of $\lambda_{3,2,1}(\mathcal{L}_6)$ to 19 [3].

At last, we will tell another variant of labelings where we answered Open Problem posted by Shashanka et al. [8] by providing lower and upper bounds of δ_1 such that $\lambda_{\delta_1,1^{k-1}}(\mathcal{L}_3) = \lambda_{1^k}(\mathcal{L}_3)$. Moreover, we show that our bound is asymptotically tight [6].

References

[1] T. Calamoneri. Optimal $L(\delta_1, \delta_2, 1)$-labeling of eight-regular grids. *Information Processing Letters*, 113(10-11):361–364, 2013.

[2] G. Chartrand, D. Erwin, F. Harary, and P. Zhang. Radio labelings of graphs. *Bulletin of the Institute of Combinatorics and its Applications*, 33:77–85, 2001.

[3] S. Das, S. C. Ghosh, and S. Nandi. Optimal $L(3, 2, 1)$-labeling of triangular lattice. *Discrete Applied Mathematics*, 2017.

[4] S. Das, S. C. Ghosh, S. Nandi, and S. Sen. A lower bound technique for radio k-coloring. *Discrete Mathematics*, 340(5):855–861, 2017.

[5] M. Kchikech, R. Khennoufa, and O. Togni. Linear and cyclic radio k-labelings of trees. *Discussiones Mathematicae Graph Theory*, 27(1):105–123, 2007.

[6] S. Nandi, N. Panigrahy, M. Agrawal, S. C. Ghosh, and S. Das. Efficient channel assignment for cellular networks modeled as honeycomb grid. *ICTCS*, pages 183–295, 2014.

[7] P. Panigrahi. A survey on radio k-colorings of graphs. *AKCE International Journal of Graphs and Combinatorics*, 6(1):161–169, 2009.

[8] M. V. S. Shashanka, A. Pati, and A. M. Shende. A characterisation of optimal channel assignments for cellular and square grid wireless networks. *Mobile Networks and Applications*, 10(1-2):89–98, 2005.