ENCODINGS OF SETS AND GRAPHS

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Hereditarily finite sets are sets whose only elements are themselves sets—all the way down to, possibly, the emptyset. In 1937 W. Ackermann proposed the following encoding of hereditarily finite sets by natural numbers:

$$\mathbb{N}_A(x) \quad = \quad \Sigma_{y \in x} 2^{\mathbb{N}_A(y)}.$$

The encoding is simple, elegant, and highly expressive for a number of reasons. On the one hand, it builds a strong bridge between two foundational mathematical structures: (hereditarily finite) sets and (natural) numbers. On the other hand, it enables the representation of the *characteristic function* of a hereditarily finite set, in terms of the usual notation for natural numbers, by sequences of binary digits: y belongs to x if and only if the $\mathbb{N}_A(y)$-th digit in the binary expansion of $\mathbb{N}_A(x)$ is equal to 1. As one would expect, the string of 0's and 1's representing $\mathbb{N}_A(x)$ is nothing but (a representation of) the characteristic function of x.

In this talk, after a short application-oriented motivating introduction, I will discuss how to connect the notions of sets and graph in both the cyclic and the acyclic cases. Starting from this connection and extending it to the case of labelled graphs, we will discuss algorithms and techniques to encode/reduce sets-and-graphs' representations. The cyclic case will be the most interesting one and will bring us to the *bisimulation* computation problem and to open problems on the extension of the Ackermann encoding for the cyclic case.

On the algorithmic side, beginning from classic work on minimization of deterministic finite state automata, we will consider coarsest relation partition problems and illustrate their connection with (numerical) encoding computation.

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