
On the Permanence of Vertices in Network Communities

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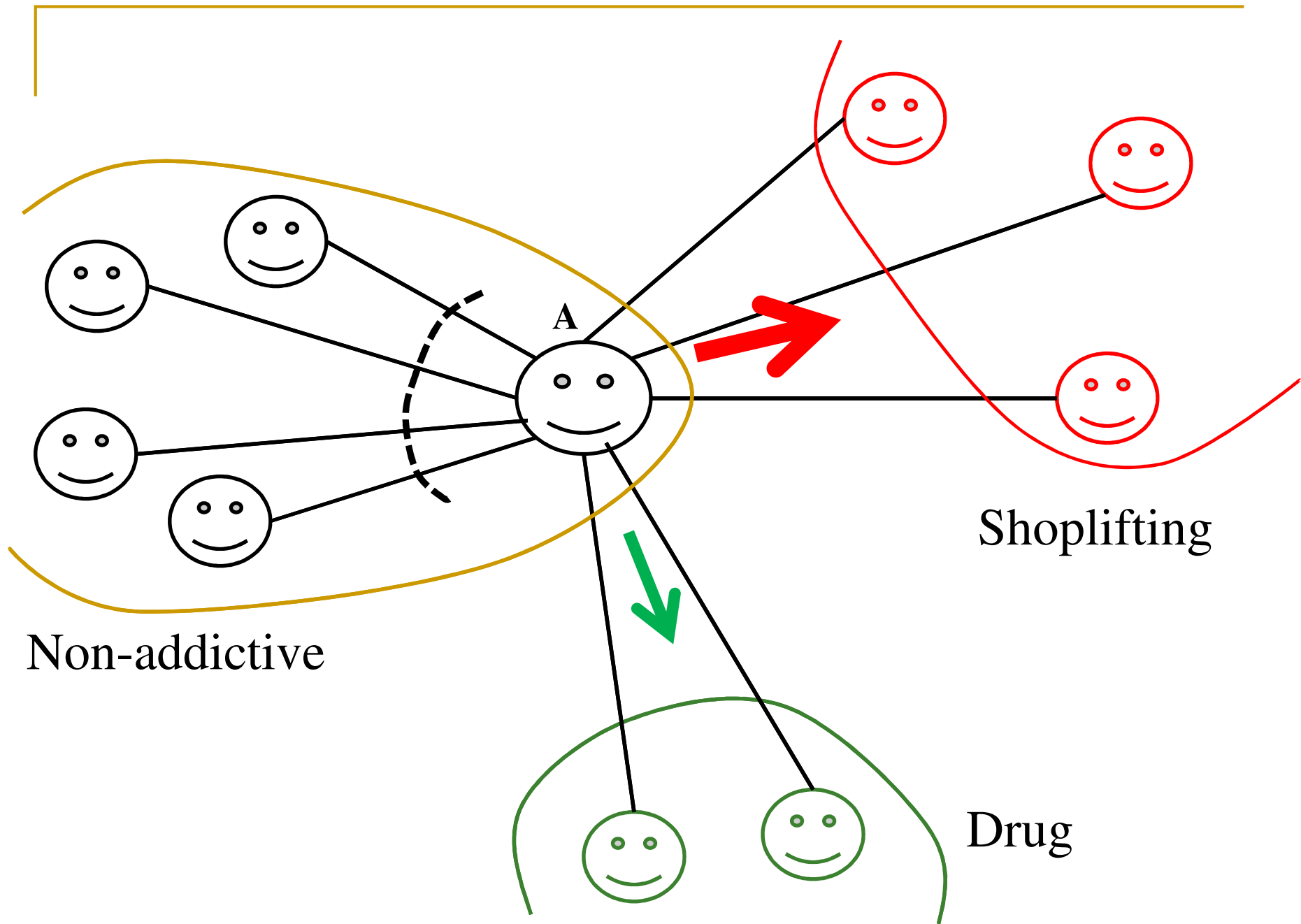
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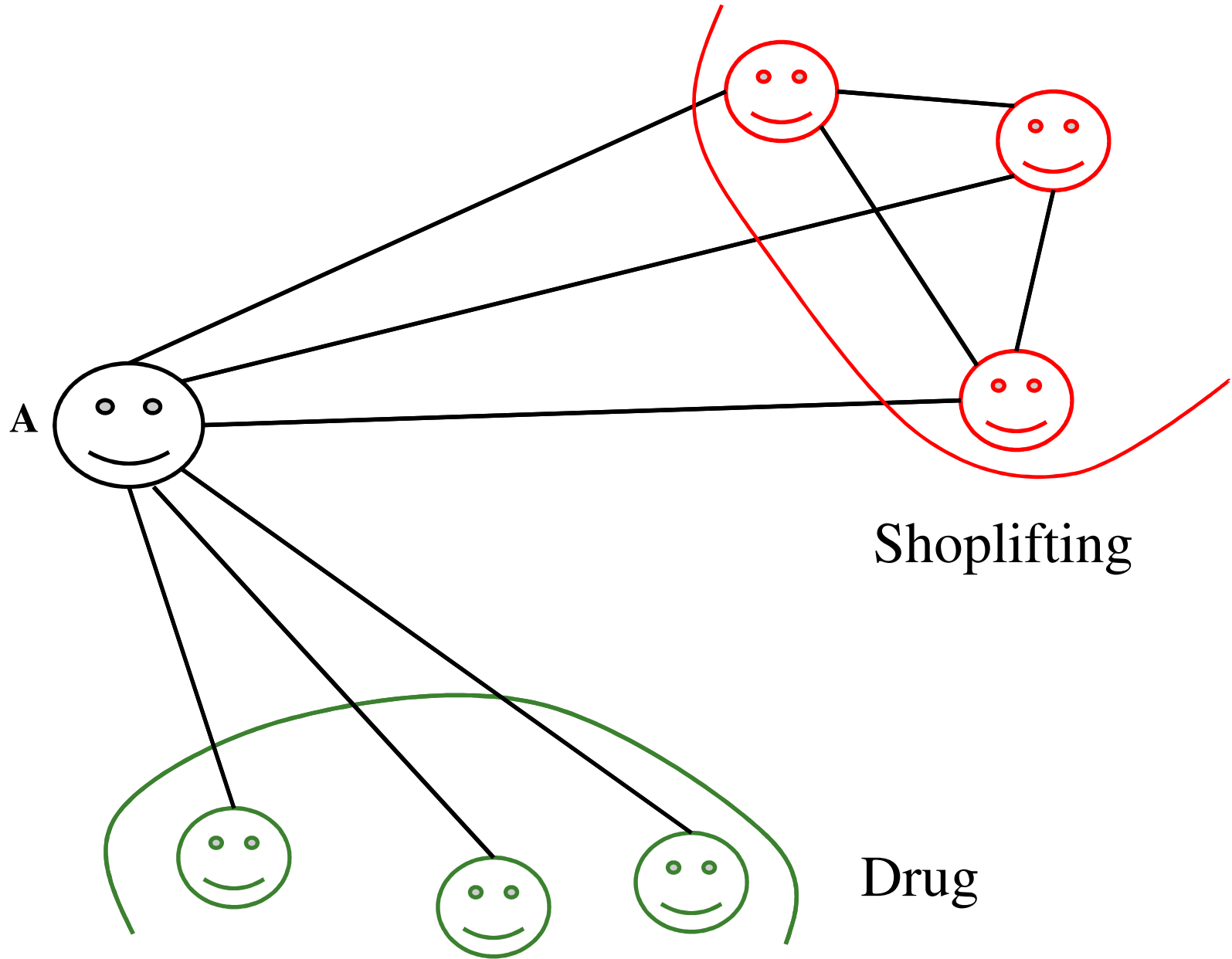


Heuristic I

Total Internal connections $>$ maximum external connections to any one of the external communities




Modularity, Conductance, Cut-ratio
consider **total external connections**



Heuristic II

Internal neighbors should be highly connected
=> high clustering coefficient among internal neighbors

 Modularity, conductance and cut-ratio
do not consider clustering coefficient



Permanence

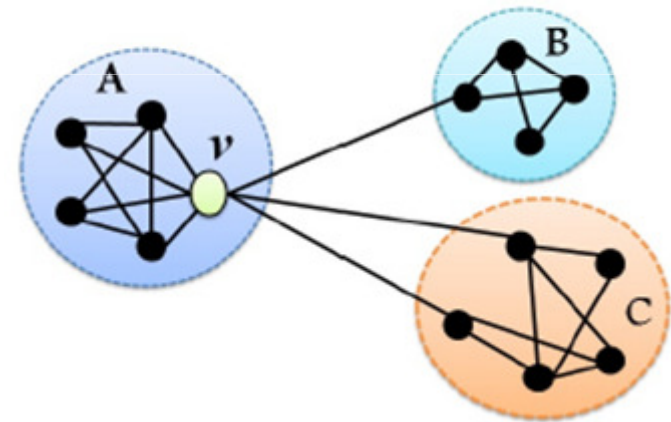
$$Perm(v) = \left[\frac{I(v)}{E_{max}(v)} \times \frac{1}{D(v)} \right] - (1 - C_{in}(v))$$

$I(v)$ = internal deg of v

$D(v)$ = degree of v

$E_{max}(v)$ = Max connection to an external neighbor

$C_{in}(v)$ = clustering coefficient of internal neighbors

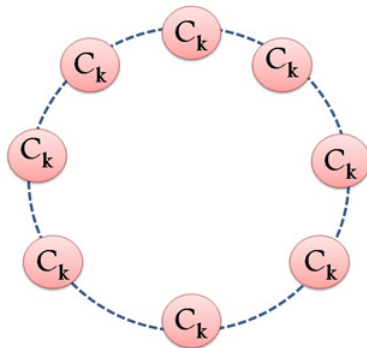


$$Perm(v) = 0.12$$

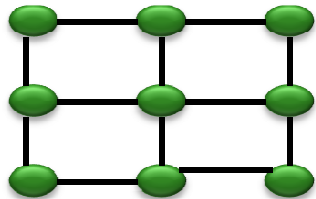
$$I(v) = 4, D(v) = 7, E_{max}(v) = 2$$

$$C_{in}(v) = 5/6$$

Permanence



Permanence ~ 1



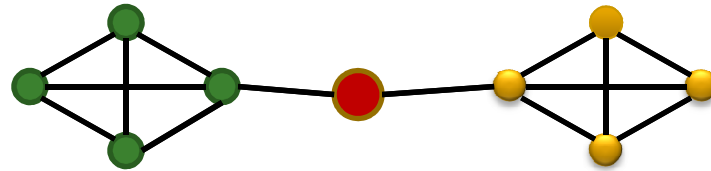
Permanence = 0

Wrong vertex-to-community
assignment

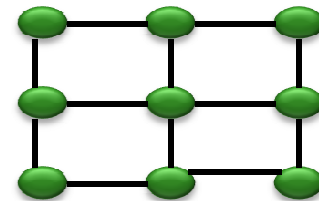
Permanence ~ -1

Research Questions

1. Assigning every vertex in a community is reasonable?



2. No one measures the **intensity of belongingness** of a vertex to a community
 - ✓ Only try to detect **best community structure** in the network
 - ✗ Never ask for whether a network possesses a strong community



Permanence answers all these questions

Test Suite of Networks

❑ Synthetic Networks

LFR networks with different values of mixing parameter (μ)

(Lancichinetti & Fortunato, PRE, 11)

❑ Real-world Networks

❑ Football Network

Nodes: teams, Edges: matches, Communities: team-conference

(Girvan & Newman, PNAS, 02)

❑ Railway Network

Nodes: station, Edges: train-connections, Communities: state/provinces

(Ghosh et al., Acta Physica, 11)

❑ Coauthorship Network

Nodes: authors, Edges: coauthorships, Communities: research field

(Chakraborty et al., ASONAM, 13)

Baseline Algorithms

❑ Modularity based

- ❑ FastGreedy (*Newman, PRE, 04*)
- ❑ Louvain (*Blondel et al, J. Stat. Mech., 08*)
- ❑ CNM (*Clauset et al, PRE, 04*)

❑ Random-walk based

- ❑ WalkTrap (*Pons & Latapy, J. Graph Algo and Appln, 06*)

❑ Compression based

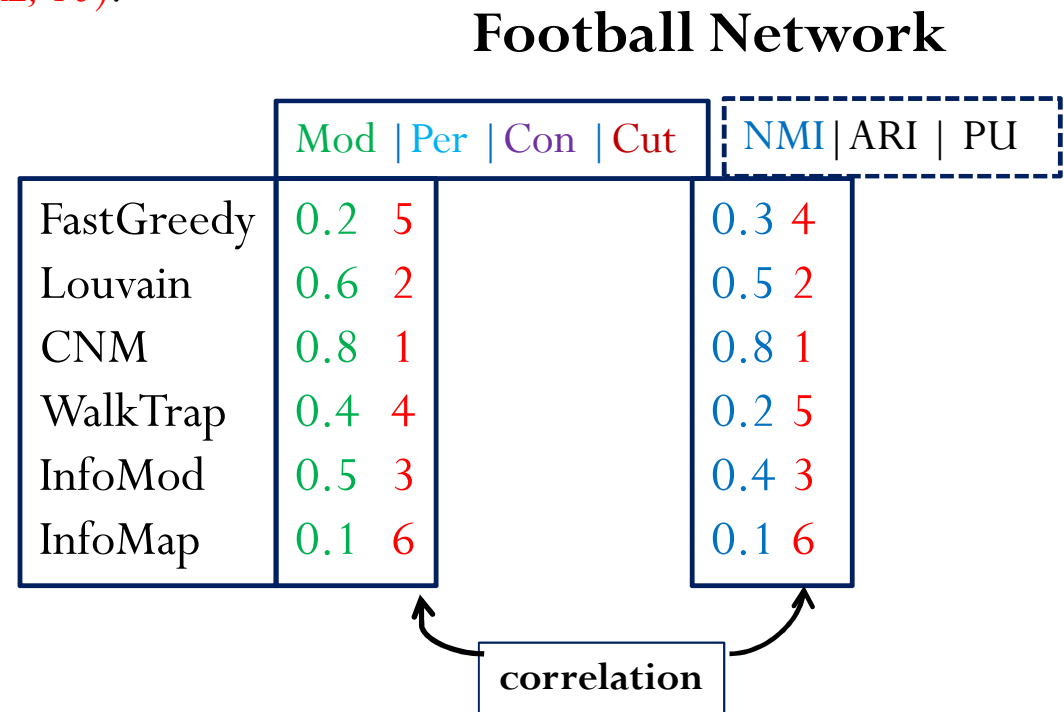
- ❑ InfoMod (*Rosvall & Bergstrom, PNAS, 07*)
 - ❑ InfoMap (*Rosvall & Bergstrom, PNAS, 08*)
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Permanence: A Better Community Scoring Function

Methodology

□ Approach *(Steinhaeuser & Chawla, PRL, 10)*:

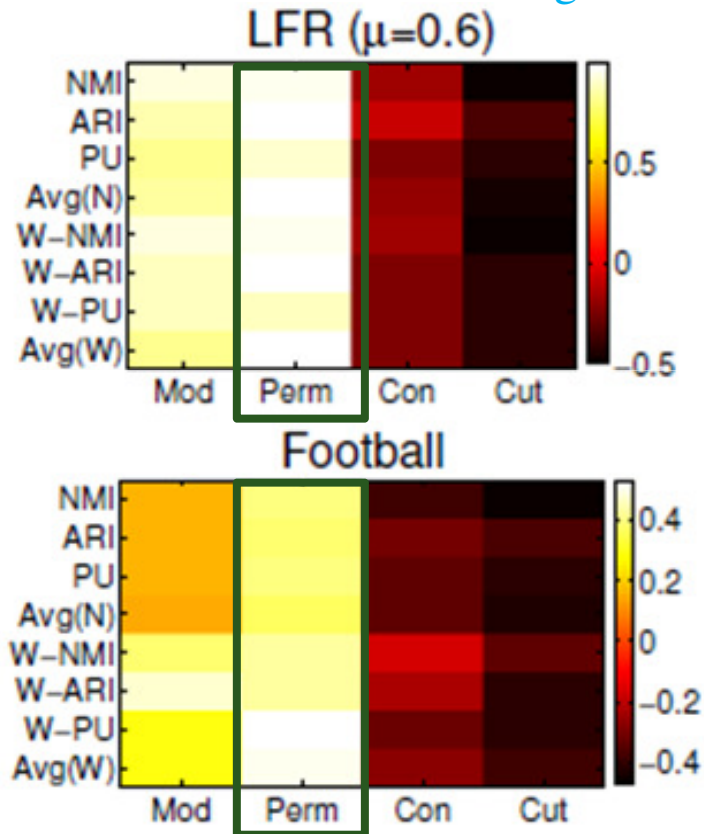
1. Consider a network
2. Run N community detection algorithms (here $N=6$)
3. Compute community scoring functions on these outputs
4. Rank the algos based on these values
5. Compare outputs with the ground-truth using validation metrics and rank algos again
6. Find rank-correlation



Intuition: Ranking of good scoring function and the validation measures should be high

Results

Lighter color is better



Networks	Modularity	Permanence	Conductance	Cut
LFR($\mu=0.1$)	0.88	0.88	0.88	0.02
LFR($\mu=0.3$)	0.61	0.74	0.72	0.28
LFR($\mu=0.6$)	0.87	0.96	-0.18	-0.44
Football	0.25	0.43	-0.29	-0.41
Railway	0.43	0.46	0.08	-0.48
Coauthorship	0.92	0.92	0.76	0.86

Table : Performance of the community scoring functions averaged over all the validation measures

Fig. : Heat maps depicting pairwise Spearman's rank correlation

Developing Community Detection Algorithm

Major Limitations

□ Limitations of optimization algorithms

- Resolution limit (*Fortunato & Barthelemy, PNAS, 07*)
 - Degeneracy of solutions (*Good et al., PRE, 10*)
 - Asymptotic growth (*Good et al., PRE, 10*)
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Community Detection Based on Maximizing Permanence

- ❑ Follow similar strategy used in Louvain algorithm (a greedy modularity maximization) (*Blondel et al., J. Stat. Mech, 07*)
 - ❑ Selecting seed nodes helps converge the process faster
 - ❑ We only consider those communities having size ≥ 3
 - ❑ Communities having size < 3 remain as singleton
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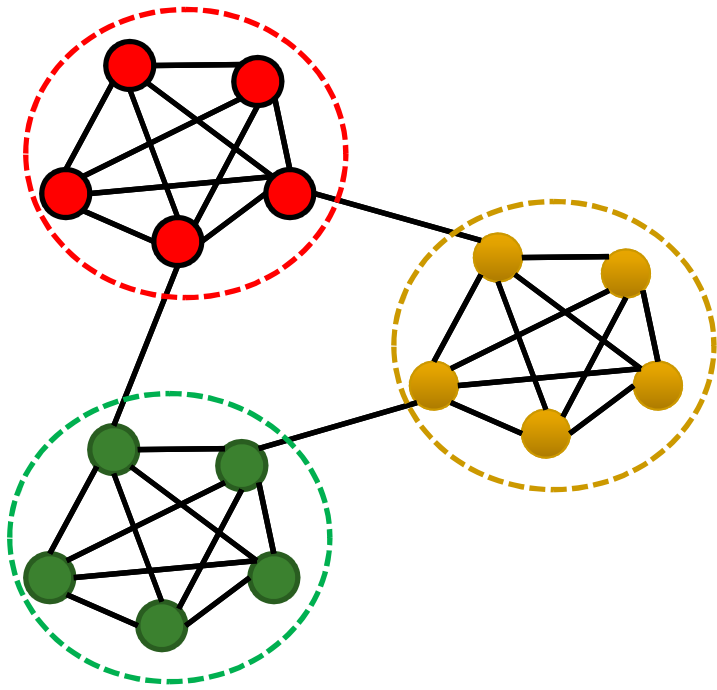
Experimental Results

Why ????

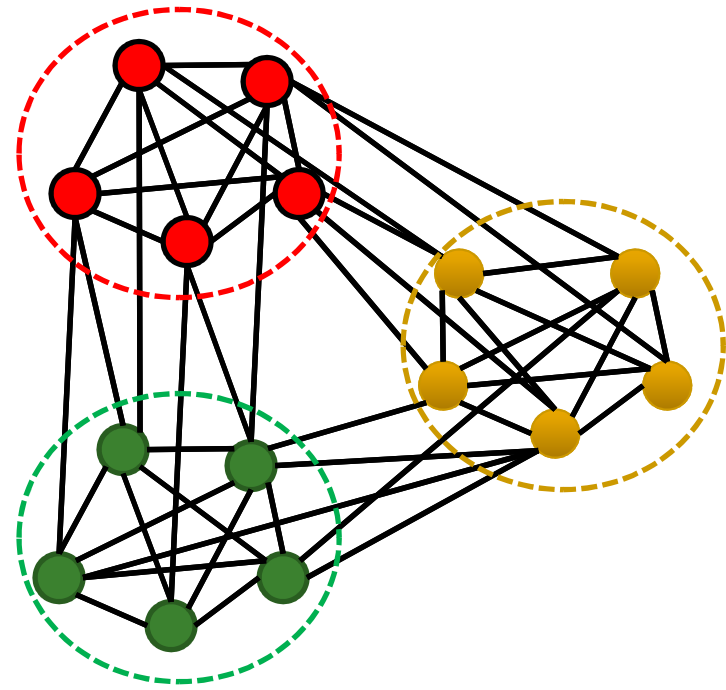
Algo	LFR ($\mu=0.1$)	LFR ($\mu=0.3$)	LFR ($\mu=0.6$)	Football	Railway	Coauthors hip
Louvain	0.02	0.00	-0.75	0.02	0.14	0.00
FastGrdy	0.00	0.87	0.02	0.01	0.37	0.14
CNM	0.14	0.40	-0.13	0.30	0.00	0.05
WalkTrap	0.00	0.00	-0.50	0.02	0.02	0.01
Infomod	0.06	0.08	-0.20	0.19	0.04	0.00
Infomap	0.00	0.00	-0.72	0.02	-0.02	0.03

Table: Differences of our algorithm with the other algorithms averaged over all validation measures

LFR ($\mu = 0.1$) vs. LFR ($\mu = 0.6$)



LFR ($\mu = 0.1$)

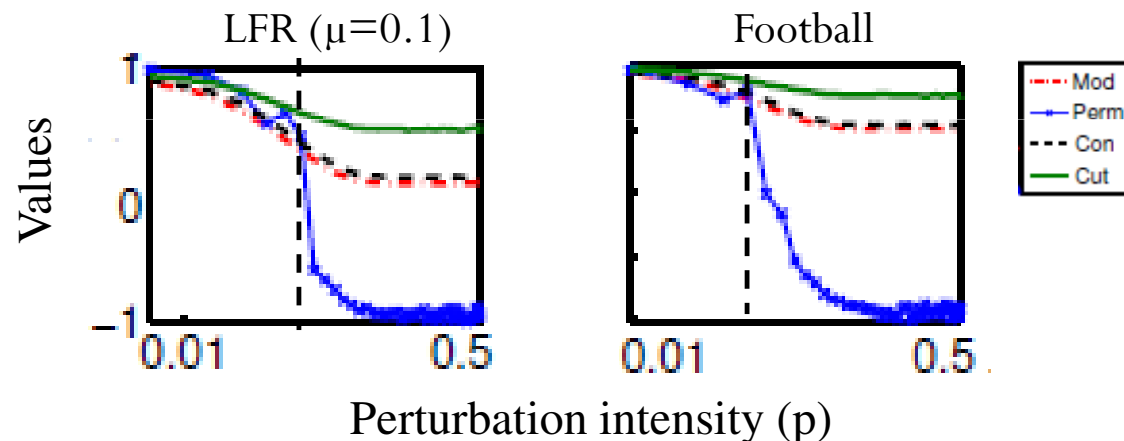


LFR ($\mu = 0.6$)

μ = Avg. ratio between the number of external connections to its degree

Permanence is Nice

- Permanence is not very sensitive to minor perturbation, but very sensitive after a certain threshold



- Permanence finds small-size communities
 - Identify **singleton** (act as junction in Railway n/w) and **small communities** (subfields in Coauthorship n/w)

Issues Related with Modularity Maximization

Resolution limit

If a vertex is **very tightly connected** to a community and **very loosely connected** to another community, highest permanence is obtained when it joins the community to which it is more connected.

Degeneracy of solution

if a vertex is **sufficiently loosely connected** to its neighbouring communities and has equal number of connections to each community, then in most cases it will remain as **singleton**, rather than arbitrarily joining any of its neighbour groups.

Asymptotic growth of value

All the parameters of parameters are **independent** of the **symmetric growth** of network size and the number of communities.

Analytical proofs: <http://cnerg.org/permanence>

Take Away

■ Permanence

- a better community scoring function
- sensitive to perturbation after a certain threshold
- indicates the eligibility of a network for community detection

■ Maximizing permanence

- a better community detection algorithm
- can detect small-size communities
- ameliorates existing limitations

□ Future work

- Recast permanence for overlapping communities
 - Recast permanence for weighted graphs
 - Recast permanence for dynamic community detection
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Thank you

<http://cnerg.org/permanence>
