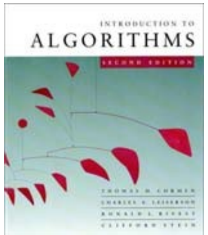
CS60020: Foundations of Algorithm Design and Machine Learning

Sourangshu Bhattacharya



The problem of sorting

Input: sequence $\langle a_1, a_2, \ldots, a_n \rangle$ of numbers.

Output: permutation $\langle a'_1, a'_2, \ldots, a'_n \rangle$ such that $a'_1 \le a'_2 \le \cdots \le a'_n$.

Example:

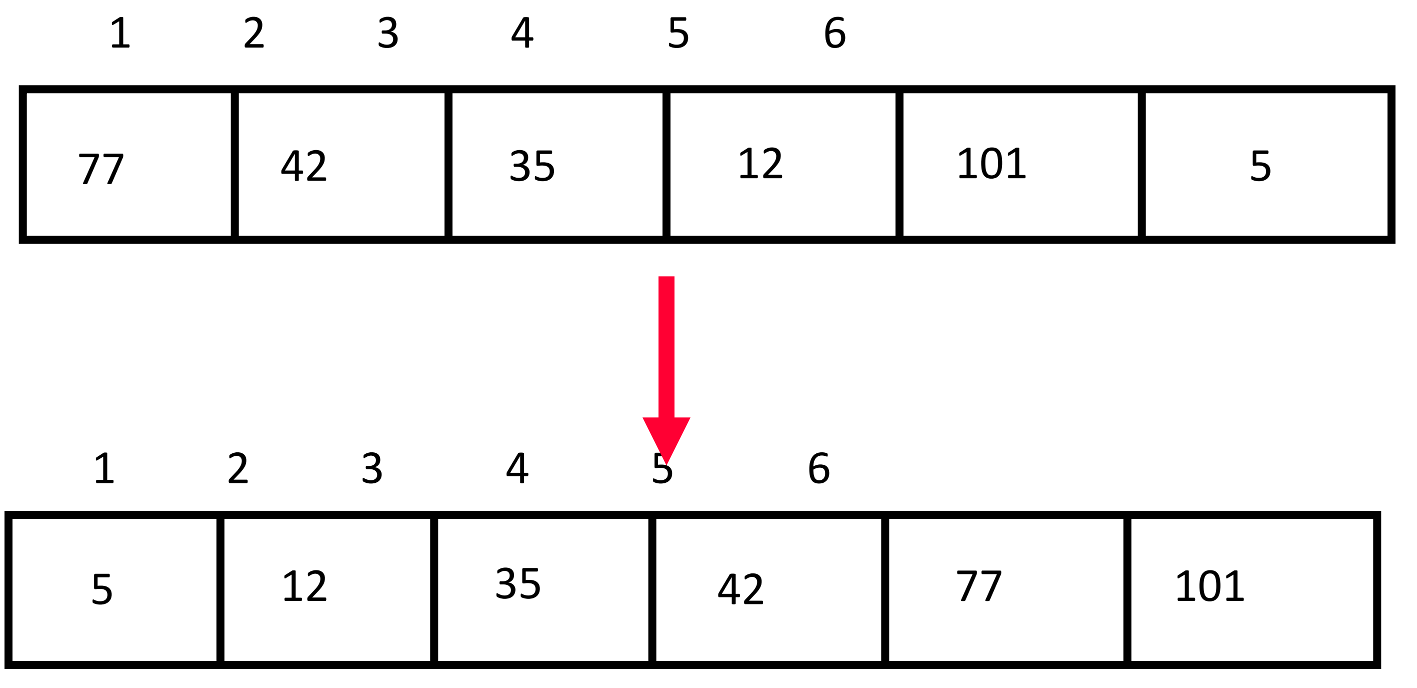
Input: 8 2 4 9 3 6

Output: 2 3 4 6 8 9

Bubble Sort

Sorting

- **Sorting takes an unordered collection and makes it an ordered one.**



"Bubbling Up" the Largest Element

- **Traverse a collection of elements**
  - **Move from the front to the end**
  - **"Bubble" the largest value to the end using pair-wise comparisons and swapping**

1	2	3	4	5	6
77	42	35	12	101	5

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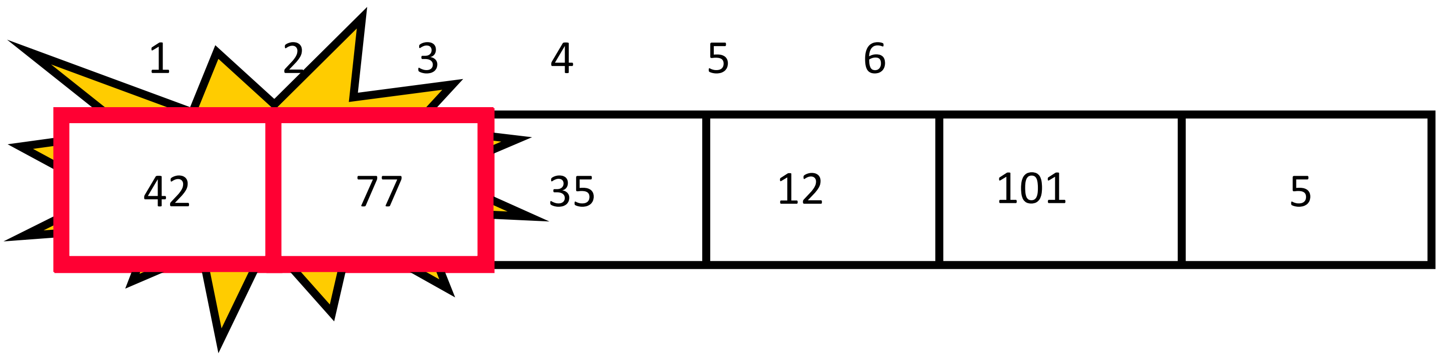

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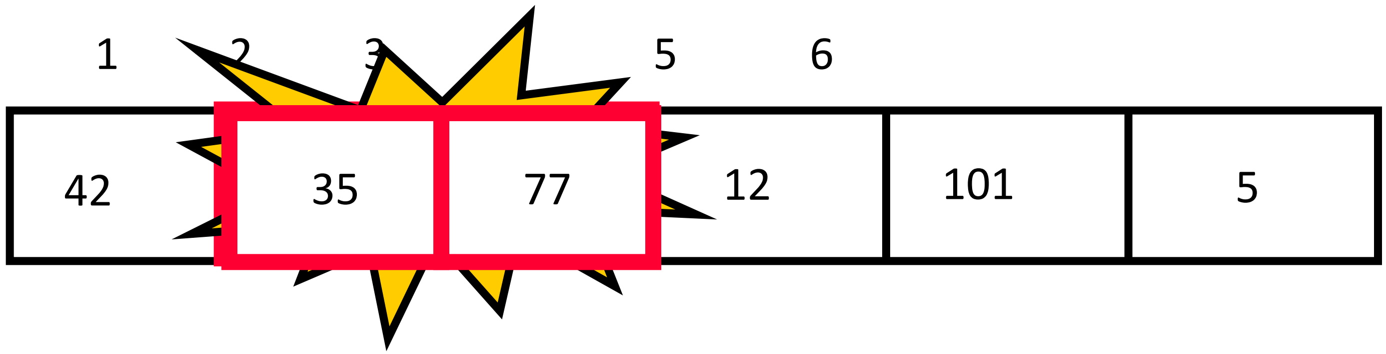

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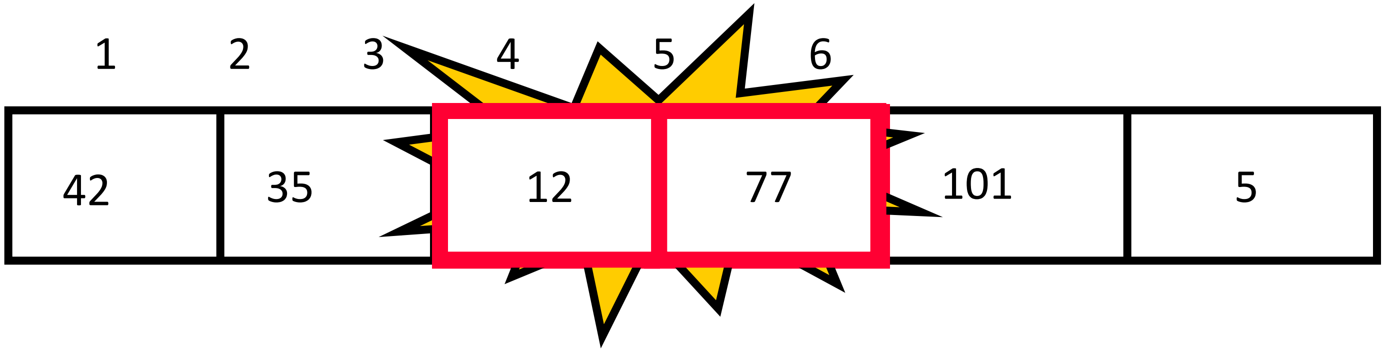

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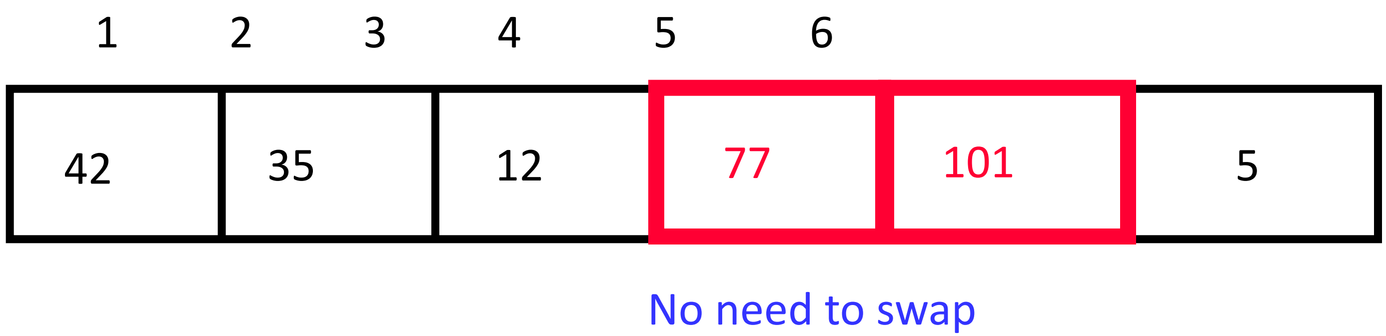

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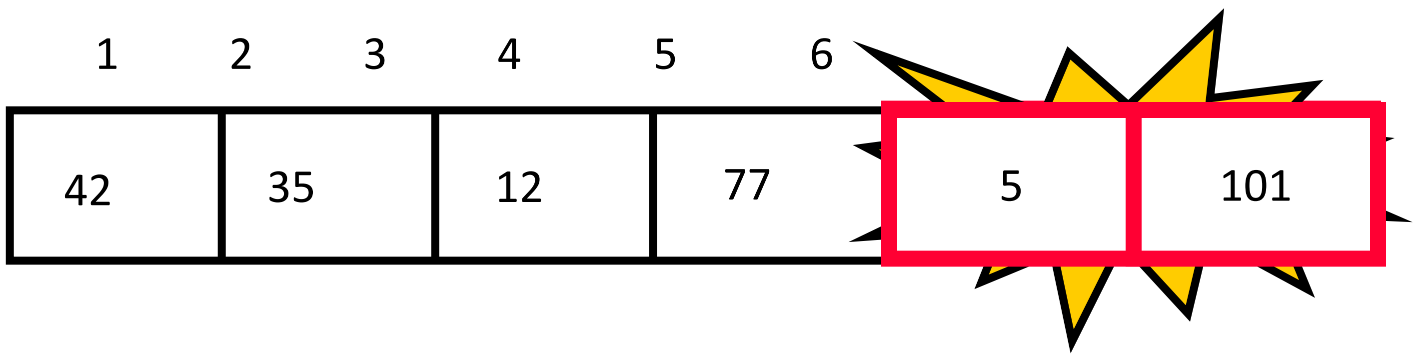

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1	2	3	4	5	6
42	35	12	77	5	101

Largest value correctly placed

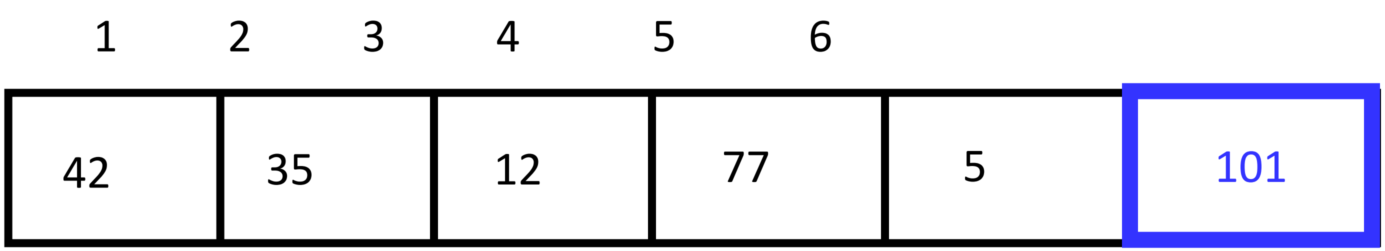
The “Bubble Up” Algorithm

```
index <- 1
last_compare_at <- n – 1

loop
  exitif(index > last_compare_at)
  if(A[index] > A[index + 1]) then
    Swap(A[index], A[index + 1])
  endif
  index <- index + 1
endloop
```

Items of Interest

- **Notice that only the largest value is correctly placed**
- **All other values are still out of order**
- **So we need to repeat this process**



Largest value correctly placed

Repeat “Bubble Up” How Many Times?

- **If we have N elements…**
- **And if each time we bubble an element, we place it in its correct location…**
- **Then we repeat the “bubble up” process N – 1 times.**
- **This guarantees we’ll correctly place all N elements.**

“Bubbling” All the Elements

N - 1

1	2	3	4	5	6
42	35	12	77	5	101

1	2	3	4	5	6
35	12	42	5	77	101

1	2	3	4	5	6
12	35	5	42	77	101

1	2	3	4	5	6
12	5	35	42	77	101

1	2	3	4	5	6
5	12	35	42	77	101

Reducing the Number of Comparisons

1	2	3	4	5	6
77	42	35	12	101	5

1	2	3	4	5	6
42	35	12	77	5	101

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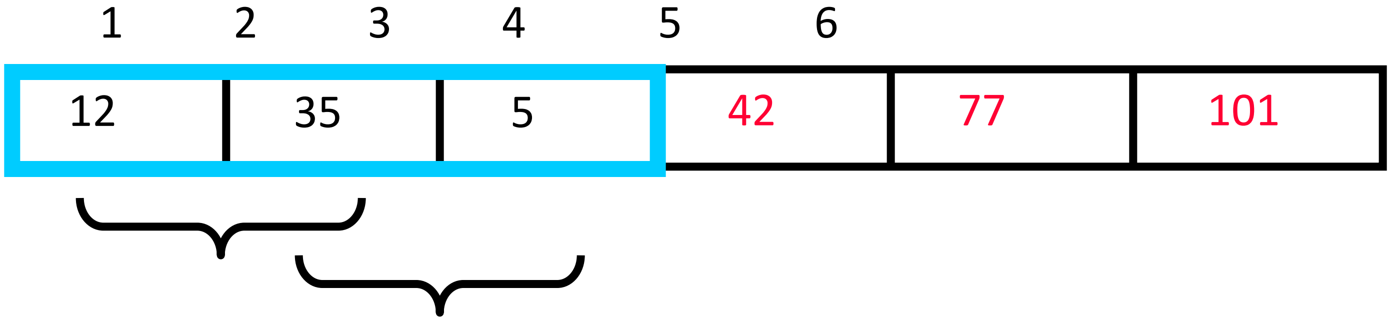
Reducing the Number of Comparisons

- **On the Nth "bubble up", we only need to do MAX-N comparisons.**

- **For example:**
  - **This is the 4th "bubble up"**
  - **MAX is 6**
  - **Thus we have 2 comparisons to do**



Putting It All Together

Bubble Sort

```
procedure Bubblesort(A isoftype in/out Arr_Type)
  to_do, index isoftype Num
  to_do <- N - 1

  loop
    exitif(to_do = 0)
    index <- 1
    loop
      exitif(index > to_do)
      if(A[index] > A[index + 1]) then
        Swap(A[index], A[index + 1])
      endif
      index <- index + 1
    endloop
    to_do <- to_do - 1
  endloop
endprocedure // Bubblesort
```

Inner loop

Outer loop

Already Sorted Collections?

- **What if the collection was already sorted?**
- **What if only a few elements were out of place and after a couple of “bubble ups,” the collection was sorted?**

- **We want to be able to detect this and “stop early”!**

1	2	3	4	5	6
5	12	35	42	77	101

Using a Boolean "Flag"

- **We can use a boolean variable to determine if any swapping occurred during the "bubble up."**

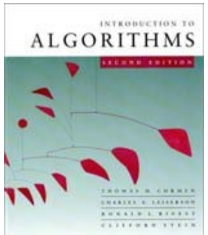
- **If no swapping occurred, then we know that the collection is already sorted!**

- **This boolean "flag" needs to be reset after each "bubble up."**

```
did_swap isoftype Boolean
did_swap <- true

loop
  exitif ((to_do = 0) OR NOT(did_swap))
  index <- 1
  did_swap <- false
  loop
    exitif(index > to_do)
    if(A[index] > A[index + 1]) then
      Swap(A[index], A[index + 1])
      did_swap <- true
    endif
    index <- index + 1
  endloop
  to_do <- to_do - 1
endloop
```

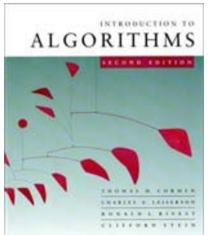
INSERTION SORT



Insertion sort

"pseudocode"

```
INSERTION-SORT (A, n)                  ◁ A[1 . . n]
    for j ← 2 to n
        do key ← A[ j]
           i ← j – 1
           while i > 0 and A[i] > key
               do A[i+1] ← A[i]
                  i ← i – 1
           A[i+1] = key
```



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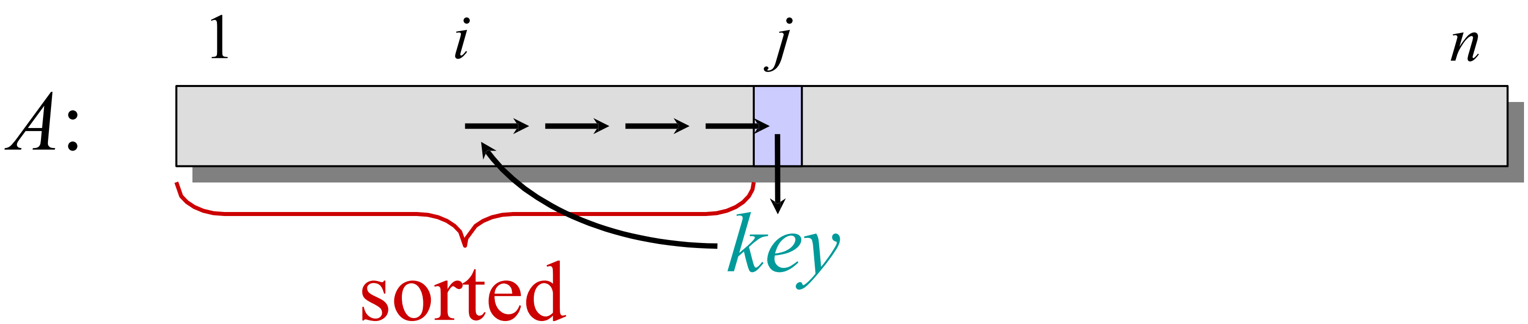
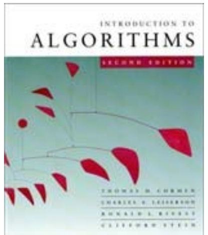
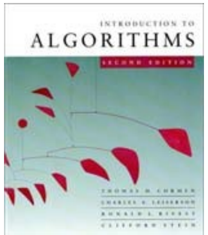




Example of insertion sort

8 2 4 9 3 6



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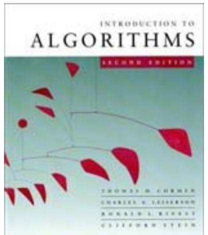
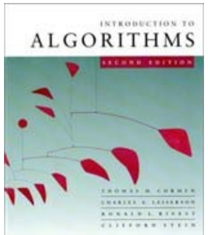


Example of insertion sort

8	2	4	9	3	6
2	8	4	9	3	6



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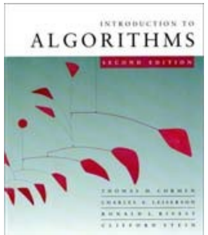

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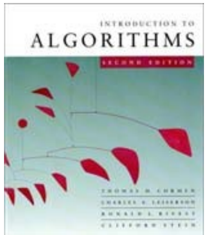

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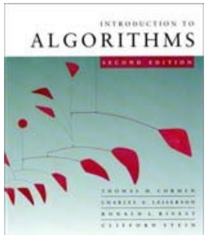
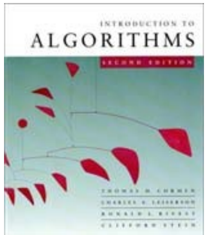


Example of insertion sort

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	4	8	9	3	6



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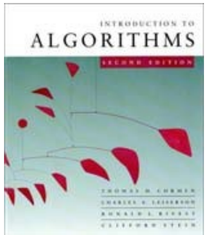

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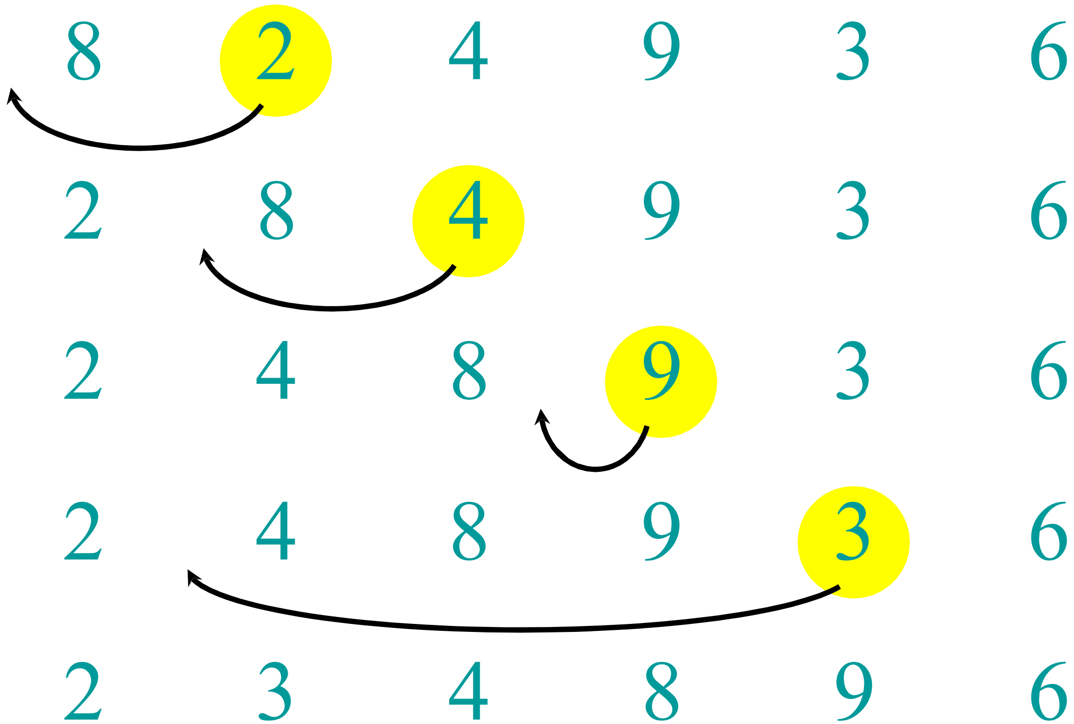
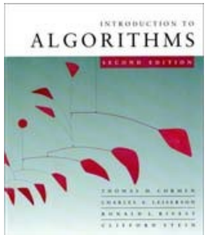




Example of insertion sort

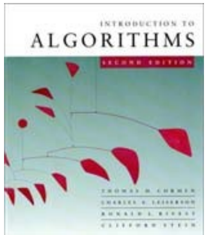
8 **2** 4 9 3 6

2 8 **4** 9 3 6

2 4 8 **9** 3 6

2 4 8 9 **3** 6

2 3 4 8 9 **6**



Example of insertion sort

8 2 4 9 3 6

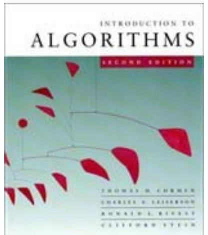
2 8 4 9 3 6

2 4 8 9 3 6

2 4 8 9 3 6

2 3 4 8 9 6

2 3 4 6 8 9 *done*



Insertion sort analysis

Worst case: Input reverse sorted.

$$T(n) = \sum_{j=2}^{n} \Theta(j) = \Theta\left(n^2\right) \qquad \text{[arithmetic series]}$$

Average case: All permutations equally likely.

$$T(n) = \sum_{j=2}^{n} \Theta(j/2) = \Theta\left(n^2\right)$$

Is insertion sort a fast sorting algorithm?

- Moderately so, for small n.
- Not at all, for large n.

Analysis

INSERTION-SORT(A)	*cost*	*times*
for $j = 2$ **to** $A.length$	c_1	n
$key = A[j]$	c_2	$n - 1$
// Insert $A[j]$ into the sorted sequence $A[1 \, . \, . \, j - 1]$.	0	$n - 1$
$i = j - 1$	c_4	$n - 1$
while $i > 0$ and $A[i] > key$	c_5	$\sum_{j=2}^{n} t_j$
$A[i + 1] = A[i]$	c_6	$\sum_{j=2}^{n} (t_j - 1)$
$i = i - 1$	c_7	$\sum_{j=2}^{n} (t_j - 1)$
$A[i + 1] = key$	c_8	$n - 1$

Comparison

Sorting Algorithm	Time Complexity			Space Complexity
	Best Case	Average Case	Worst Case	Worst Case
Bubble Sort	**$O(N)$**	**$O(N^2)$**	**$O(N^2)$**	**$O(1)$**
Selection Sort	**$O(N^2)$**	**$O(N^2)$**	**$O(N^2)$**	**$O(1)$**
Insertion Sort	**$O(N)$**	**$O(N^2)$**	**$O(N^2)$**	**$O(1)$**