Constant Approximation Algorithms for Guarding Simple Polygons using Vertex Guards

Pritam Bhattacharya

Department of Computer Science and Engineering,
Indian Institute of Technology, Kharagpur

pritam.bhattacharya@cse.iitkgp.ernet.in

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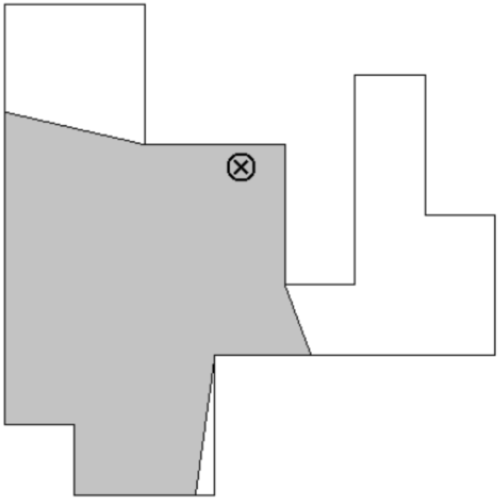
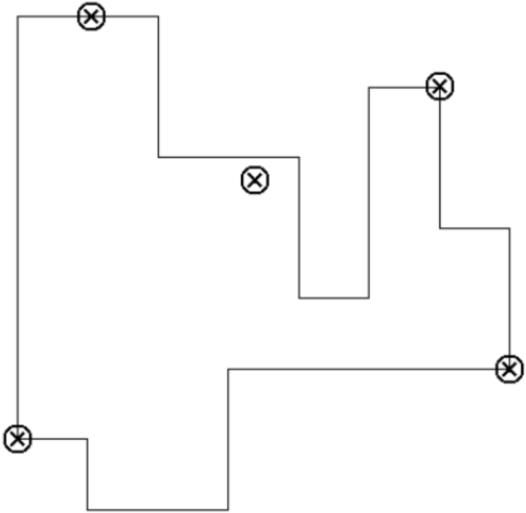
Visible Region from a Single Guard / Camera



Coverage by Multiple Guards / Cameras



Polygons and Visibility

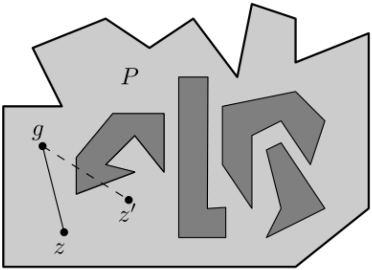


Figure: Polygon with holes

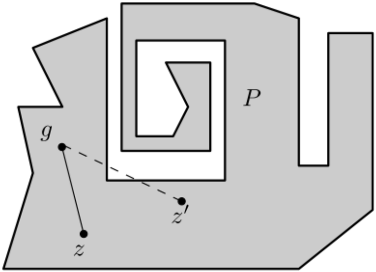


Figure: Polygon without holes

Definition (Visibility of a Point)

Any point $z \in P$ is said to be *visible* from another point $g \in P$ if the line segment zg lies wholly in the interior of P.

Art Gallery Problem

The art gallery problem (AGP) is about finding the least number of guards that are necessary to ensure that an art gallery is fully guarded, assuming that a guard's field of view covers 360° as well as an unbounded distance.

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An art gallery can be abstracted as an n-sided polygon P (with or without holes), and guards as special points within P.

Art Gallery Problem - Types of Guards

Different types of guards considered in Art Gallery Problem:

(1) Point guards (stationary)

(2) Perimeter guards (stationary)

(3) Vertex guards (stationary)

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(2) Perimeter guards (stationary)

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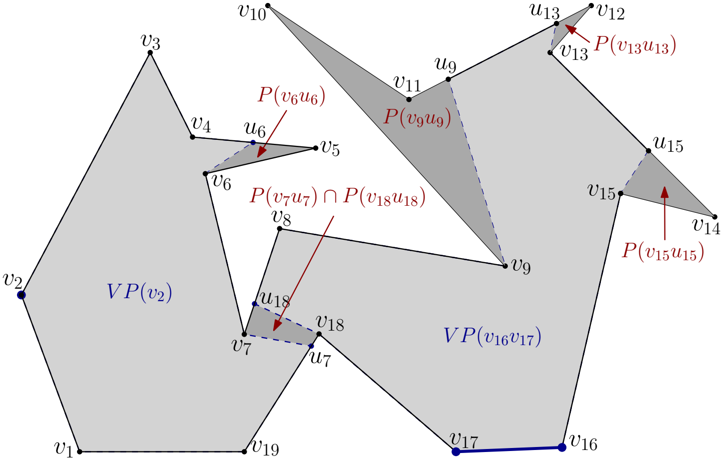
(4) Edge guards (mobile)

(5) Segment guards (mobile)

Visibility Polygon of a Guard



Sufficient Number of Guards

Theorem (Chvatal (1975), Fisk (1978))

For guarding a simple polygon with n vertices, $\lfloor \frac{n}{3} \rfloor$ point guards or vertex guards are sufficient and sometimes necessary.

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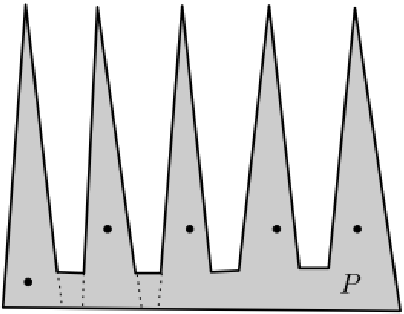

Figure: A polygon where $\lfloor \frac{n}{3} \rfloor$ point guards or vertex guards are necessary.

Art Gallery Problem - Computational Hardness

Definition (Decision Version of the Art Gallery Problem)

Given a polygon P and a number k as input, can the polygon P be guarded with k or fewer guards?

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Hardness results known for the decision version of AGP:

- Proved to be NP-complete for vertex guards (Lee and Lin)
- Proved to be NP-hard for point guards (Aggarwal)
- Very recently proved to be ETR-complete (Abrahamsen, Adamaszek & Miltzow)
- Proved to be APX-hard (Eidenbenz, Stamm and Widmayer), implying that no PTAS can exist for AGP.
- Specifically for polygons with holes, AGP cannot be approximated to within a factor of $\Omega(\ln n)$ (Eidenbenz, Stamm and Widmayer).

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- They provide provable guarantees on the distance of the returned solution to the optimal one.
- Typically the returned solution is always guaranteed to be within a multiplicative factor of the optimal solution, and this multiplicative factor is referred to as the *approximation ratio*.

Motivation for our Work

Theorem (Ghosh, 1987)

By discretization of the polygon and a subsequent reduction to the Set Cover problem, we can obtain a $\mathcal{O}(\log n)$*-approximation algorithm for guarding polygons using vertex and edge guards.*

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Theorem (Eidenbenz, Stamm & Widmayer, 1998)

For polygons with holes, there cannot exist a polynomial time algorithm for AGP with an approximation ratio better than $((1-\epsilon)/12)\ln n$ for any $\epsilon > 0$, unless $NP \subseteq TIME(n^{\mathcal{O}(\log\log n)})$.

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Conjecture (Ghosh, 1987)

There exist polynomial time algorithms with a constant approximation ratio for guarding polygons without holes using vertex guards or edge guards.

Our Results - Polygons Weakly Visible from an Edge

Definition (Weak Visibility Polygon)

A polygon P is said to be a *weakly visible* if it has a special edge uv such that, for every point p within P, p is visible from some point q on uv.

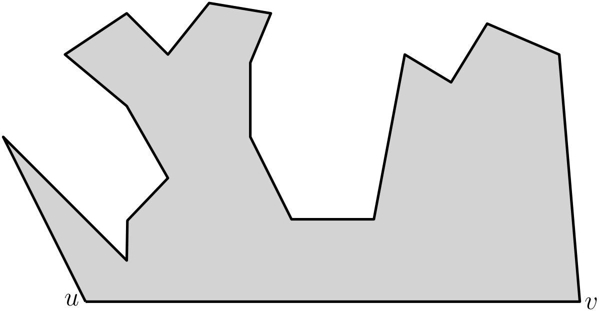

Figure: An example of a polygon weakly visible from an edge uv

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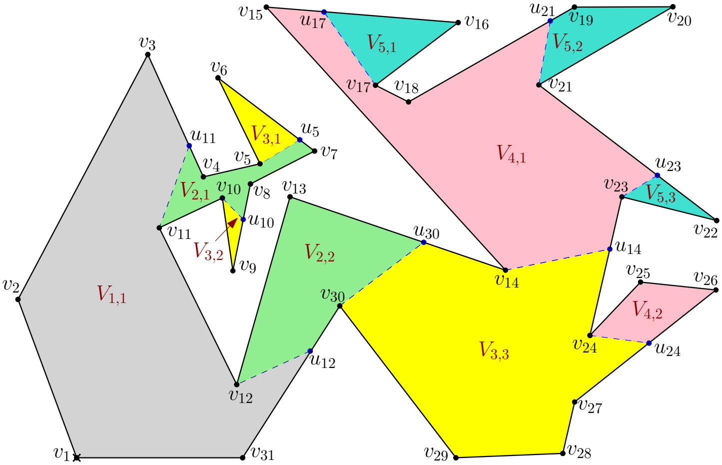
We obtained:

- A **6-approximation algorithm**, which has running time $\mathcal{O}(n^2)$, for guarding polygons that are weakly visible from an edge and contain no holes, using vertex guards.

Our Results - General Simple Polygons



Our Results - General Simple Polygons

Theorem

A set S of vertex (edge) guards for guarding all vertices of a simple polygon P can be computed in $\mathcal{O}(n^4)$ time, such that $|S| \leq 18 \times |G_{opt}|$, where G_{opt} is an optimal vertex (edge) guard cover for all vertices of P.

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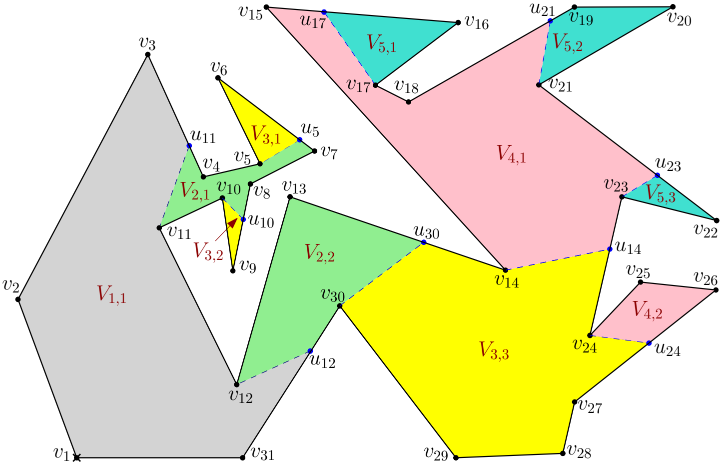

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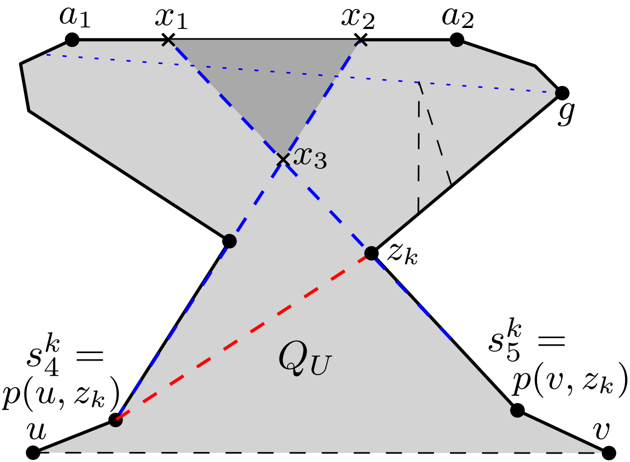

Figure: All vertices are visible from $p(u, z_k)$ or $p(v, z_k)$, but the triangle $x_1x_2x_3$ is invisible.

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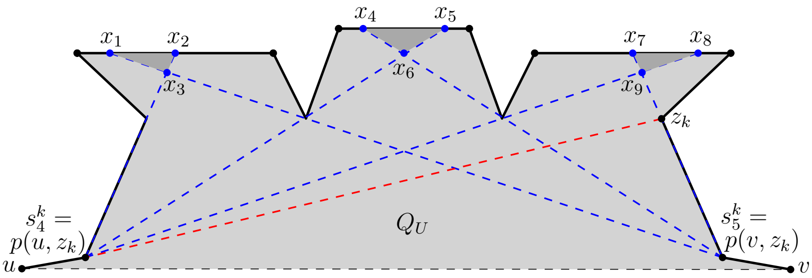


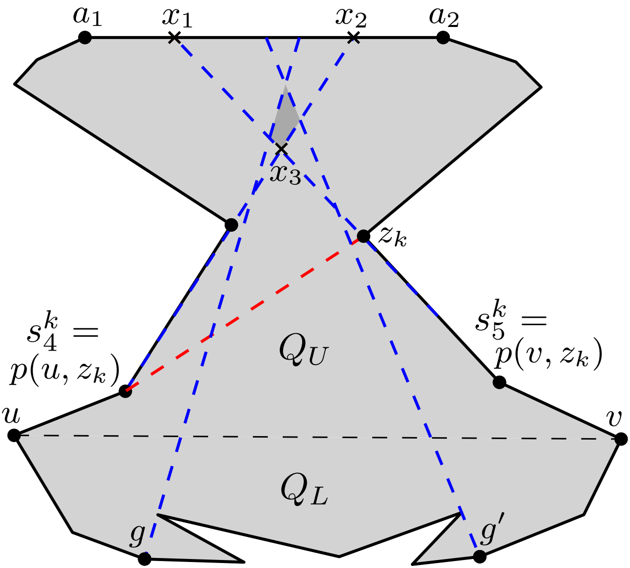
Figure: Multiple invisible cells exist within the polygon that are not visible from the guards placed at $p(u, z_k)$ and $p(v, z_k)$.

Our Results - General Simple Polygons

Theorem

A set S of vertex (edge) guards for guarding the entire boundary of a simple polygon P can be computed in $\mathcal{O}(n^5)$ *time, such that* $|S| \leq 18 \times |G_{opt}|$*, where* G_{opt} *is an optimal vertex (edge) guard cover for the entire boundary of P.*

Our Results - General Simple Polygons



Our Results - General Simple Polygons

Theorem

A set S of vertex (edge) guards for guarding the entire interior of a simple polygon P can be computed in $\mathcal{O}(n^5)$ time, such that $|S| \leq 27 \times |G_{opt}|$, where G_{opt} is an optimal vertex (edge) guard cover for the entire interior of P.

Future Directions and Open Problems

- Can we obtain similar approximation algorithms when using perimeter guards rather than vertex / edge guards?

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- Can we obtain similar approximation algorithms when using perimeter guards rather than vertex / edge guards?
- What about the case where the polygon P under question can have vertices inserted or deleted dynamically?
- Can we obtain similar results if we consider one-reflection or k-modems visibility instead of direct visibility?

Thank you!