Practice Problems: Minimum Cost Flow

January 19, 2024

1. Consider the standard linear programming relaxation for the minimum cost perfect matching in a bipartite graph taught in the class and its dual linear program. Let $x \in \mathbb{R}^{\mathcal{E}}$ be a feasible solution of the primal linear program. Prove that x is a primal optimal solution if and only if there exists a dual solution $y \in \mathbb{R}^{\mathcal{V}}$ such that

$$\forall \{u, v\} \in \mathbb{E} : x_{uv} > 0 \Rightarrow y_u + y_v = c_{uv}$$

2. Consider the standard linear programming relaxation for the transshipment problem taught in the class and its dual linear program. Let $x \in \mathbb{R}^{\mathcal{E}}$ be a feasible solution of the primal linear program. Prove that x is a primal optimal solution if and only if there exists a dual solution $y \in \mathbb{R}^{\mathcal{V}}$ such that

$$\forall (u, v) \in \mathbb{E} : x_{uv} > 0 \Rightarrow y_v = y_u + p_{uv}$$

   Does your proof goes through had the problem been mincost flow instead of transshipment?

3. Show that the minimum cost flow problem "reduces" to the transshipment problem in polynomial time.

4. Show a feasible standard linear program has an optimal solution if its dual is feasible. Use the fact that the optimal value of a feasible standard linear program is either $-\infty$ or finite.

5. Show that the feasibility of a transshipment problem instance can be checked by solving one $s - t$ max flow instance.

6. Show that the $s - t$ max flow problem "reduces" to the mincost flow problem.

7. Show that the $s - t$ shortest path problem "reduces" to the mincost flow problem.