Indian Institute of Technology Kharagpur
Dept. Computer Science and Engineering

Subject No.: CS31005 Subject Name: Algorithms II
Total marks: 60 Duration: 2 hours

Mid Semester Examination

Answer all questions.

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1. (a) [8 points] Prove or disprove: For every positive integer n, there exists a sequence of insertion, extract-min, and decrease-key operations starting from an initially empty Fibonacci heap so that the height of the resulting Fibonacci heap is at least $\frac{n}{10}$.

   (b) [7 points] Show two cascading cut operations using an example of a sequence of insert, decrease-key, and extract-min operations performed on an initially empty Fibonacci heap.

2. (a) [7 points] Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ be an undirected and unweighted graph. For $\mathcal{X} \subseteq \mathcal{V}$, we define $\delta(\mathcal{X})$ to be the capacity of the cut $(\mathcal{X}, \mathcal{V} \setminus \mathcal{X})$. Then show the following for every two subsets $\mathcal{A}, \mathcal{B} \subseteq \mathcal{V}$

$$\delta(\mathcal{A}) + \delta(\mathcal{B}) \geqslant \delta(\mathcal{A} \cup \mathcal{B}) + \delta(\mathcal{A} \cap \mathcal{B})$$

   (b) [8 points] We are given a flow network $\mathcal{G} = (\mathcal{V}, \mathcal{E}, c : \mathcal{E} \longrightarrow (1, \infty))$ and a maximum flow $f : \mathcal{E} \longrightarrow \mathbb{R}_{>0}$ in that network. Suppose the capacity of an edge $e \in \mathcal{E}$ is decreased by 1. Give an $\mathcal{O}(m + n)$ time algorithm to compute a maximum flow of the modified network.

3. (a) [8 points] Suppose you are given a black-box access to three subroutines, namely, max, min, and median, each running in linear time, to compute respectively the max, min, and median of an array of n (not necessarily distinct) integers. Design a worst-case linear time algorithm to find the i^{th} smallest integer amongst n elements where i is given as an input. Your algorithm may call the max, min, and median subroutines but should not compare any pair of elements of the array (outside the subroutine calls to

max, min, and median). Prove the correctness and worst-case time complexity (that should include the time complexity of the subroutines called) of your algorithm.

(b) [5 points] In the median-finding algorithm SELECT taught in the class, what would be the running time of the algorithm if the group sizes of elements were 7 instead of 5?

(c) [2 points] In the median-finding algorithm SELECT taught in the class, what would be the running time of the algorithm if the group sizes were 3 instead of 5?

4. We have two strings $P[1, ..., m]$ over $\Sigma \cup \{?\}$ and $T[1, ..., n]$ over Σ.

(a) [5 points] Design an algorithm to check if $m = n$ and, for all $i \in \{1, \ldots, n\}$, $P[i] =?$ or $P[i] = T[i]$.

(b) [10 points] Design an algorithm to find all integers s such that, for all $i \in \{1, \ldots, m\}$, $P[i] =?$ or $P[i] = T[s + i]$. What is the running time of your algorithm?