

# Problems: Intractability 2, Approximation Algorithms

Palash Dey  
Indian Institute of Technology, Kharagpur

October 9, 2023

1. Design a polynomial time algorithm for 2SAT problem.
2. Show that 3SAT many-to-one reduces to 3-Colorability of graphs.
3. Show that Subset Sum many-to-one reduces to Knapsack.
4. Show that Subset Sum many-to-one reduces to Partition.
5. Show that Hamiltonian Cycle many-to-one reduces to Hamiltonian Path.
6. Show that DNF-SAT is polynomial time solvable.
7. Show that Subgraph Isomorphism is NP-complete.
8. Consider the task of scheduling  $n$  jobs into  $m$  identical machines. Each job  $i$  needs to run  $t_i$  time in any machine to complete. A job once started to run in a machine cannot be stopped midway. A schedule is an assignment  $\mathcal{A} : [n] \rightarrow [m]$ . The load  $\ell_j$  of machine  $j$  in a schedule  $\mathcal{A}$  is  $\sum_{i \in \mathcal{A}^{-1}(j)} t_i$ . The makespan of a schedule  $\mathcal{A}$  is  $\max_{j \in [m]} \ell_j$ . Show that computing a schedule which achieves the minimum makespan is NP-complete. Design a simple 2 factor approximation algorithm for this problem. Change the algorithm suitably to improve the approximation guarantee to  $\frac{3}{2}$ .