

# Problems: Fibonacci Heaps

Sudeshna Kolay  
Indian Institute of Technology, Kharagpur

August 16, 2023

1. Analyse the running time of the Dijkstra's algorithm when the priority queue being used in a Fibonacci Heap instead of a Binary Heap. Assume that the number of vertices in the graph is  $n$  and the number of edges  $m$ .
2. Suppose in the Fibonacci Heap, we allowed an operation `ChangeKey`, which takes as input the pointer to a node in a Fibonacci Heap  $H$  and a new key value  $k$ , and changes  $x.key$  to  $k$ . Note that  $k$  could be larger than, equal to or less than the current key value. Analyse the amortised running time of this operation.
3. [based on CLRS book] Suppose that we generalise the cascading-cut rule to cut a node  $x$  from its parent as soon as it loses its  $k^{\text{th}}$ , for some integer constant  $k$ . For what value of  $k$  is  $D(n) = O(\log n)$ ? What are the definitions of the standard operations on Fibonacci heaps with this new definition? Give an argument with the accounting method for the amortized costs of each operation.