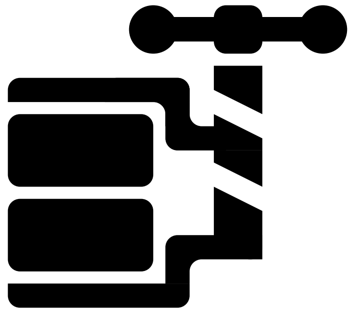
Iterative Compression



Slides by Prof. Neeldhara Misra

Input A graph G = (V,E) with n vertices, m edges, and k.

Is there a vertex cover of size at most k? **Question**

Vertex Cover

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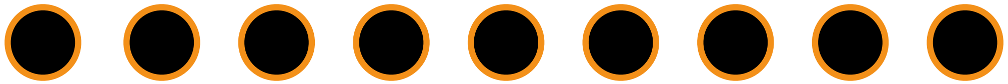


A vertex cover of size (k+1).

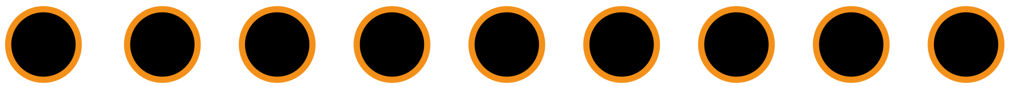
Vertex Cover

Input

A graph G with a vertex cover of size k+1.

Is there a vertex cover of size at most k?

Question



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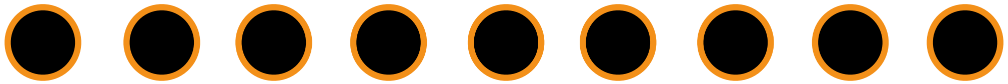

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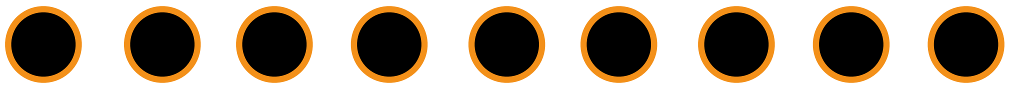

A vertex cover of size (k+1).

Let us "guess" how a vertex cover of size at most k interacts with this one.

Vertex Cover

Input

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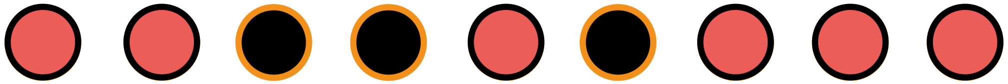
Is there a vertex cover of size at most k?

Question



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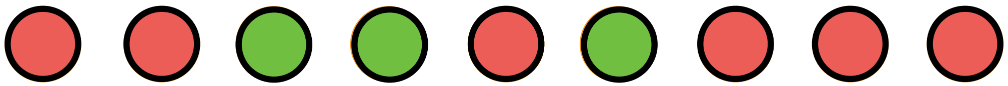

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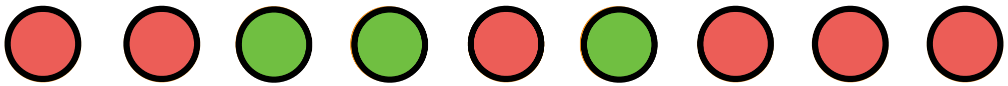

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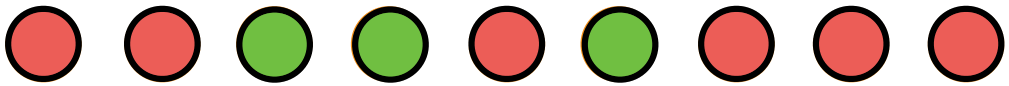


A vertex cover of size (k+1).

Can there be an edge between two red vertices?

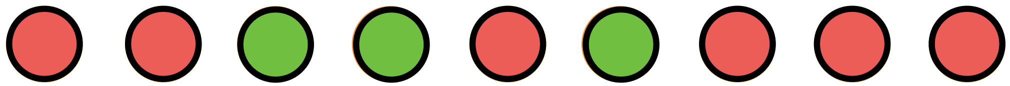
Vertex Cover

Input

A graph G with a vertex cover of size k+1.

Is there a vertex cover of size at most k?

Question



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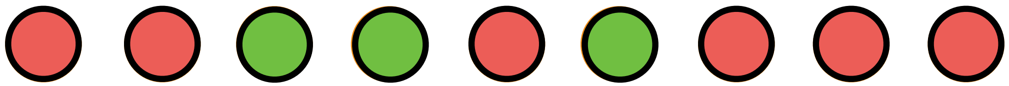

A vertex cover of size (k+1).

Now we need to make up for the work that the red vertices were doing.

Vertex Cover

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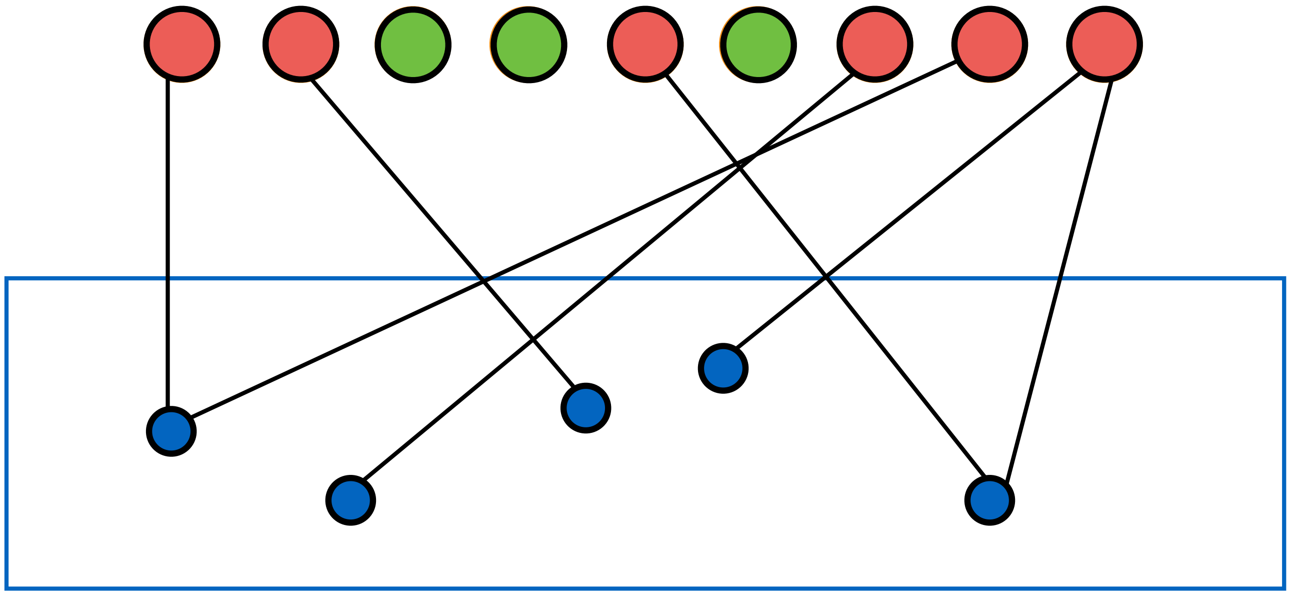
A graph G with a vertex cover of size k+1.

Question

Is there a vertex cover of size at most k?



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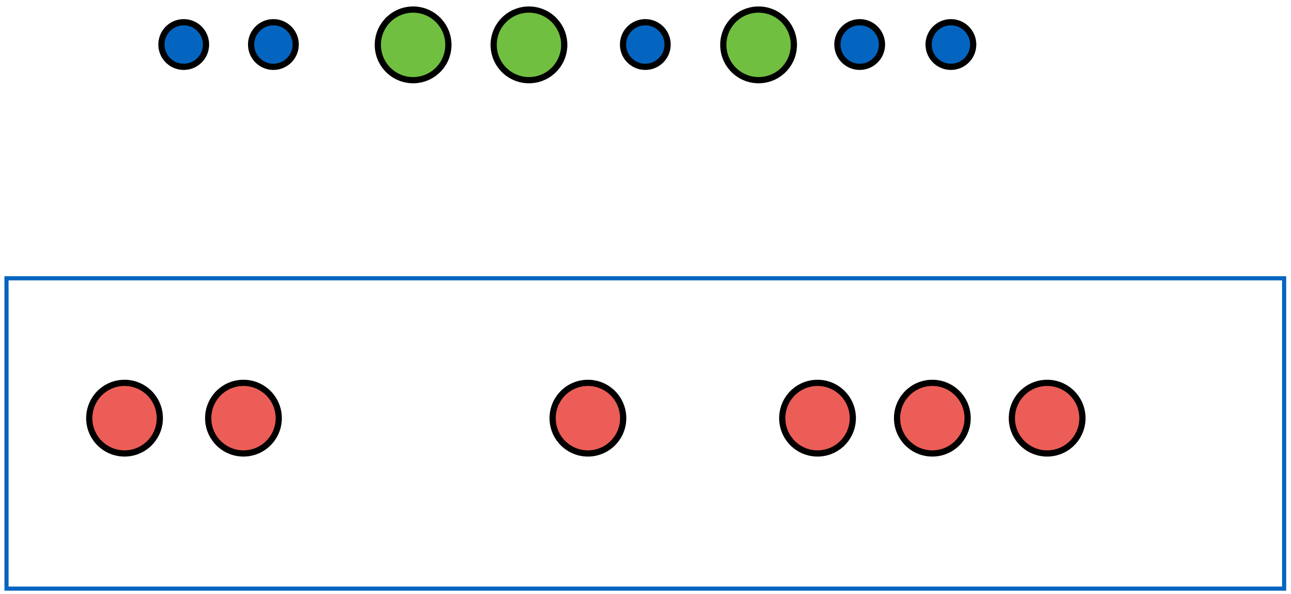

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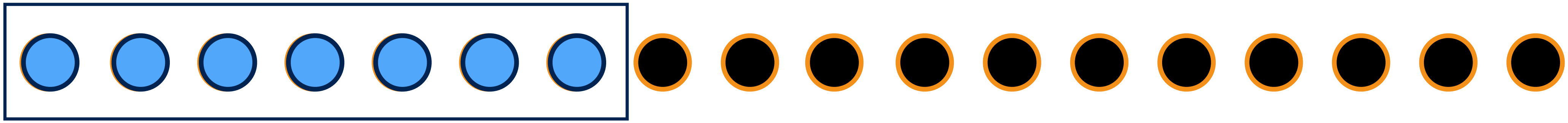

The first k+2 vertices in G.

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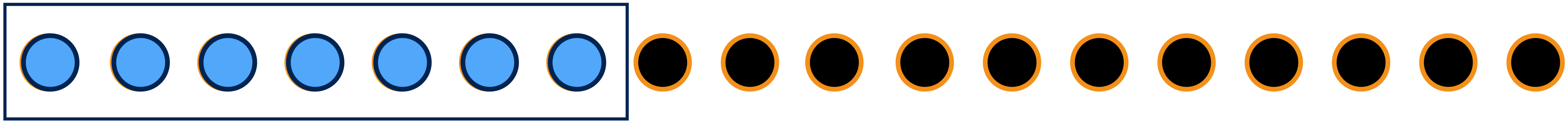

The first k+2 vertices in G. It has an easy vertex cover of size k+1.

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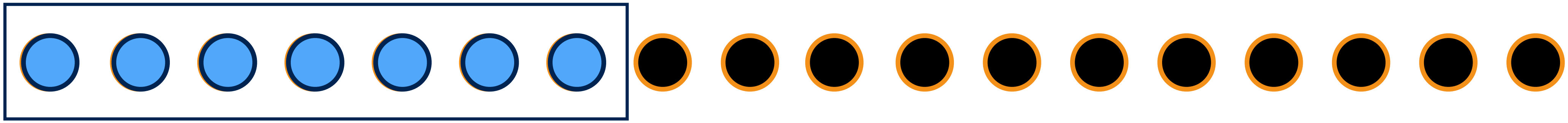

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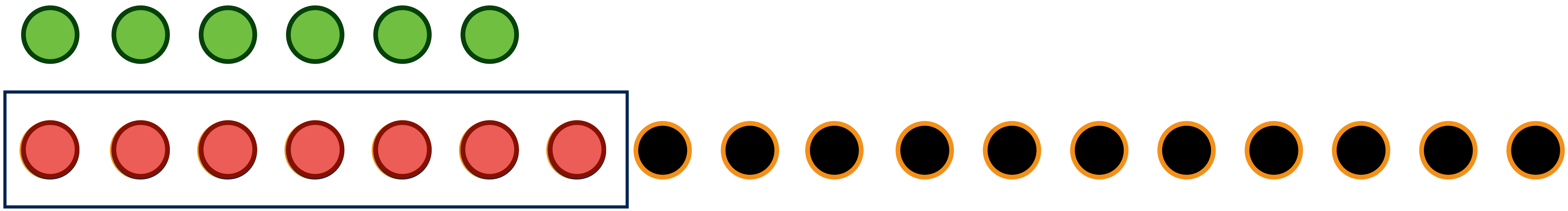

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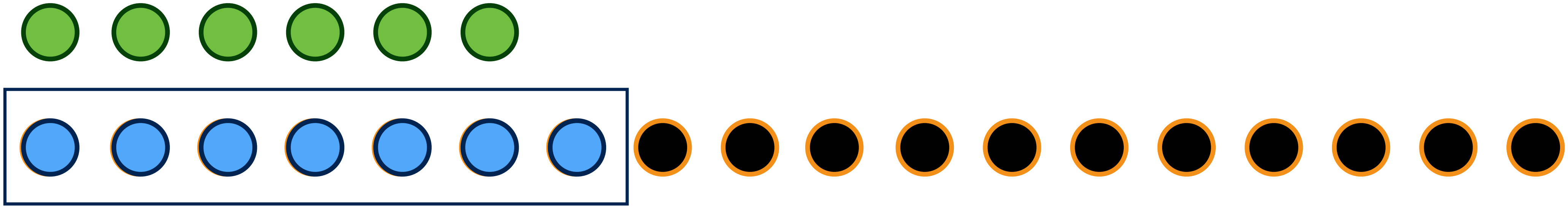

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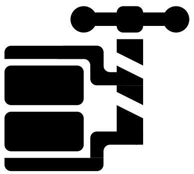

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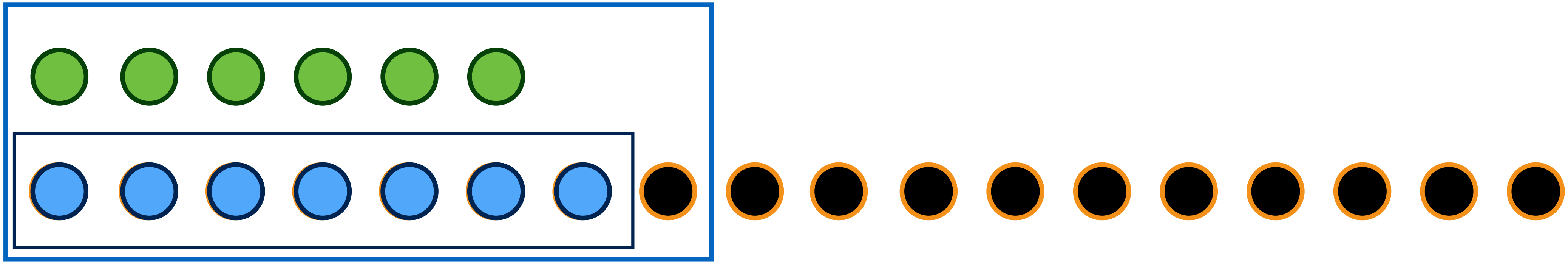

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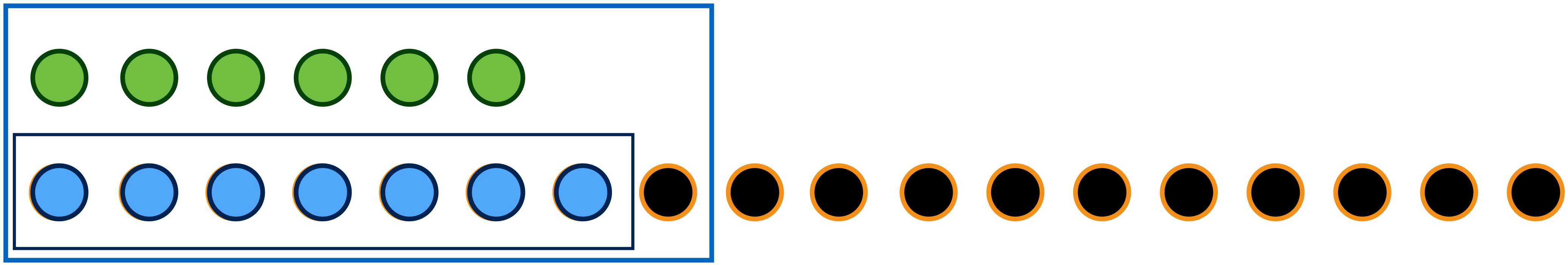

The first k+2 vertices in G. It has an easy vertex cover of size k+1.

Compress again… rinse, repeat.

Vertex Cover

Input A graph $G = (V,E)$ with n vertices, m edges, and k.

Is there a subset of vertices S of size at most k that intersects all the edges? Question

We have a $O^*(2^k)$ algorithm for Vertex Cover.

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Is there a subset of vertices S of size at most k that intersects all the edges? **Question**

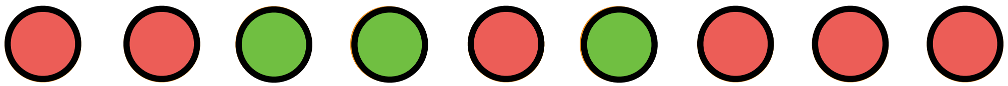
We have a $O^*(2^k)$ algorithm for Vertex Cover.

☺

Vertex Cover

Input A graph on n vertices, m edges and an integer k.

Is there a subset of at most k vertices whose removal makes the graph acyclic? Question



A Feedback Vertex Set of size (k+1).

Let us "guess" how a FVS of size at most k interacts with this one.

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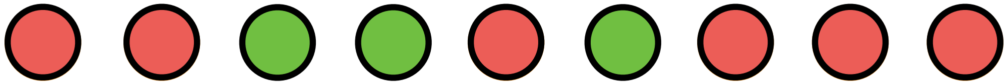

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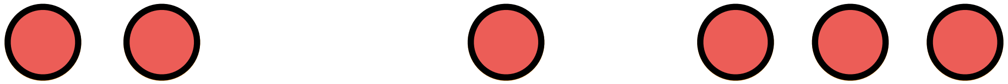

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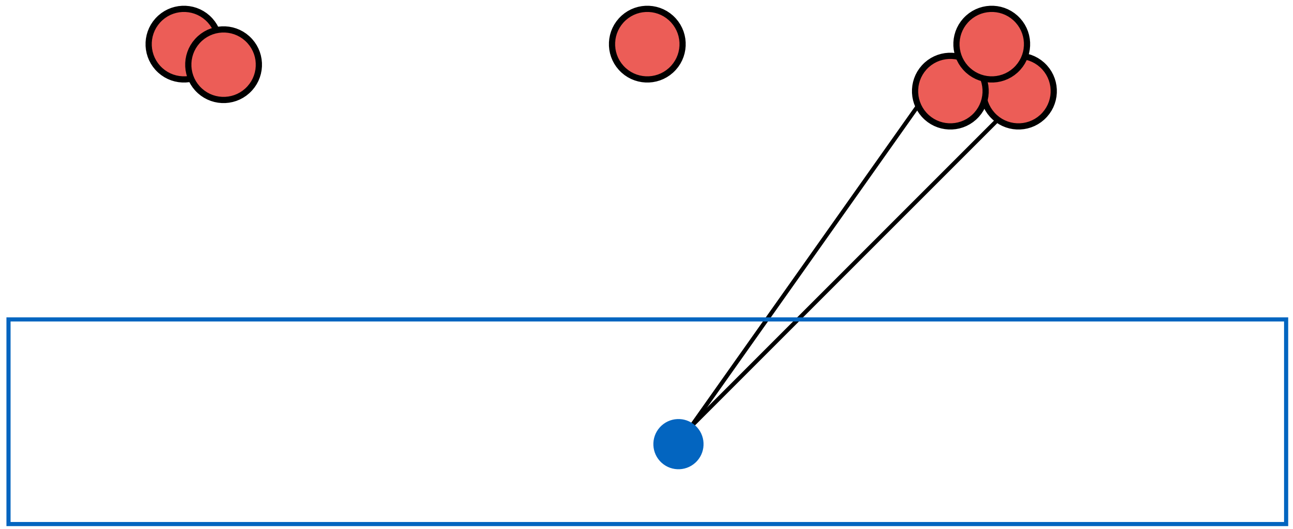

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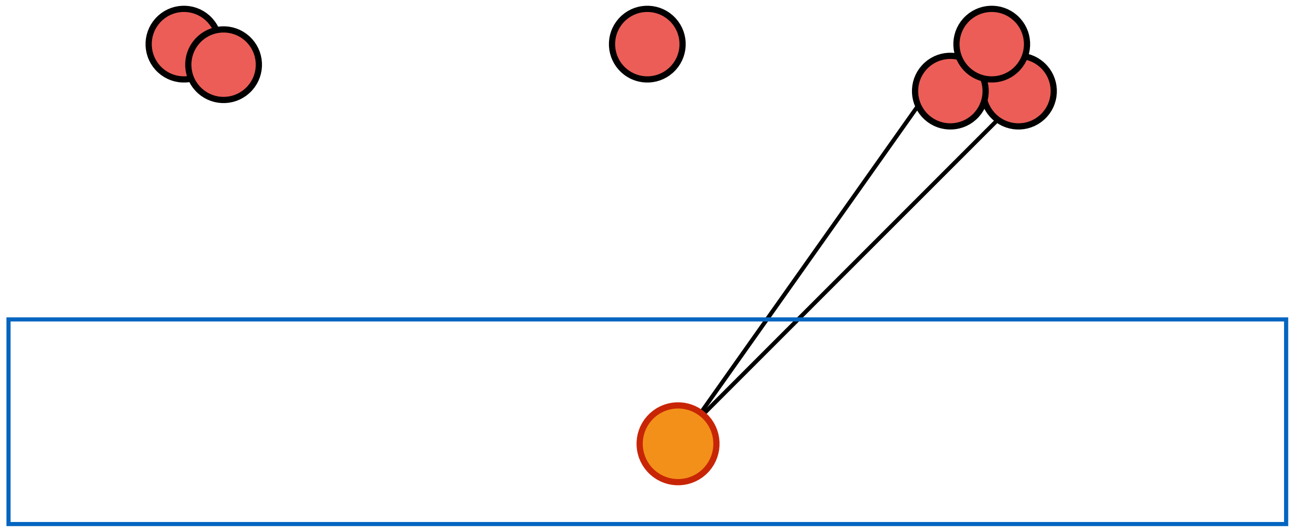

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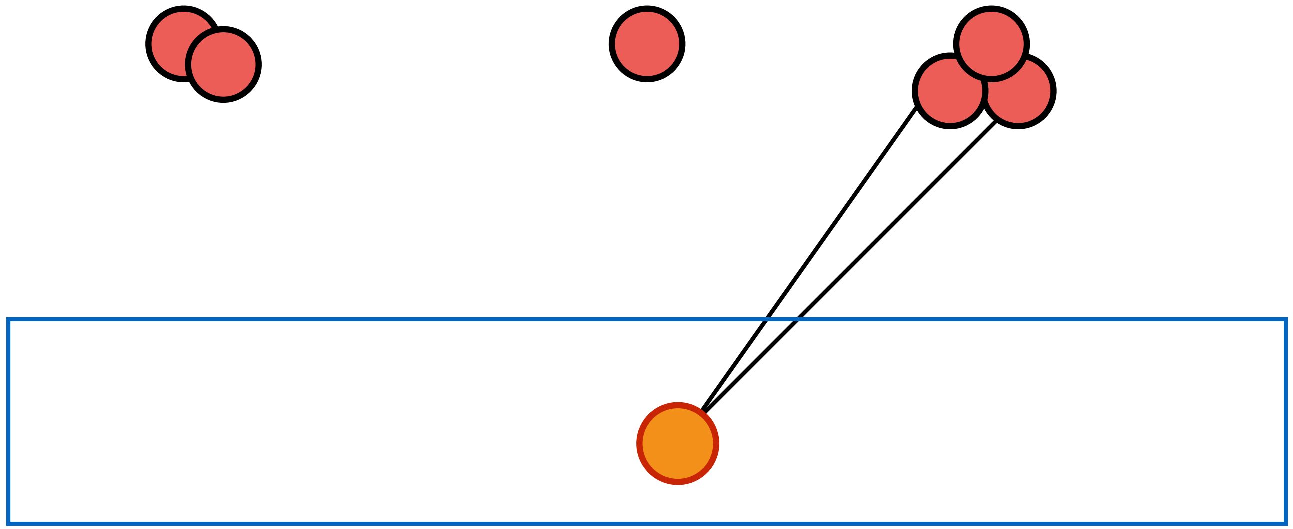

A vertex with two neighbors in the same component is forced.

FEEDBACK VERTEX SET

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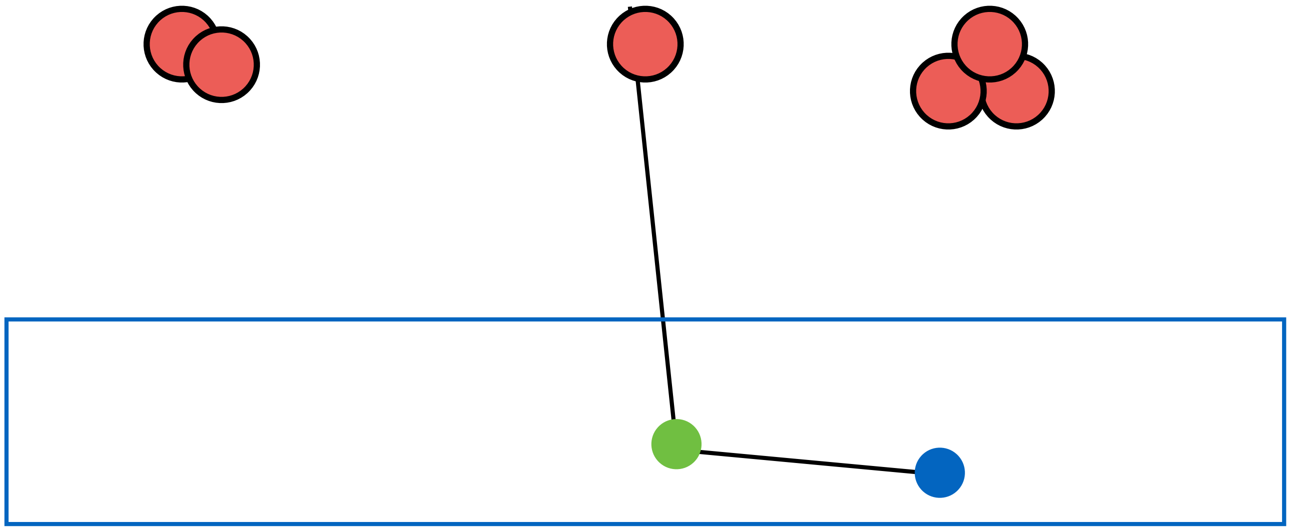

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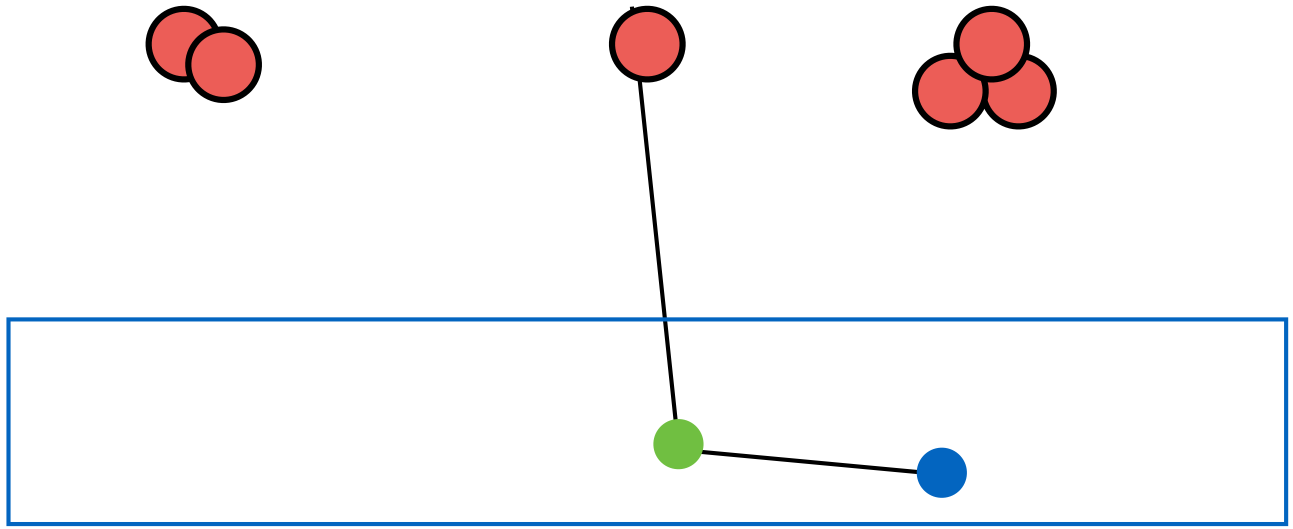

A leaf that has exactly one neighbor above can be preprocessed.

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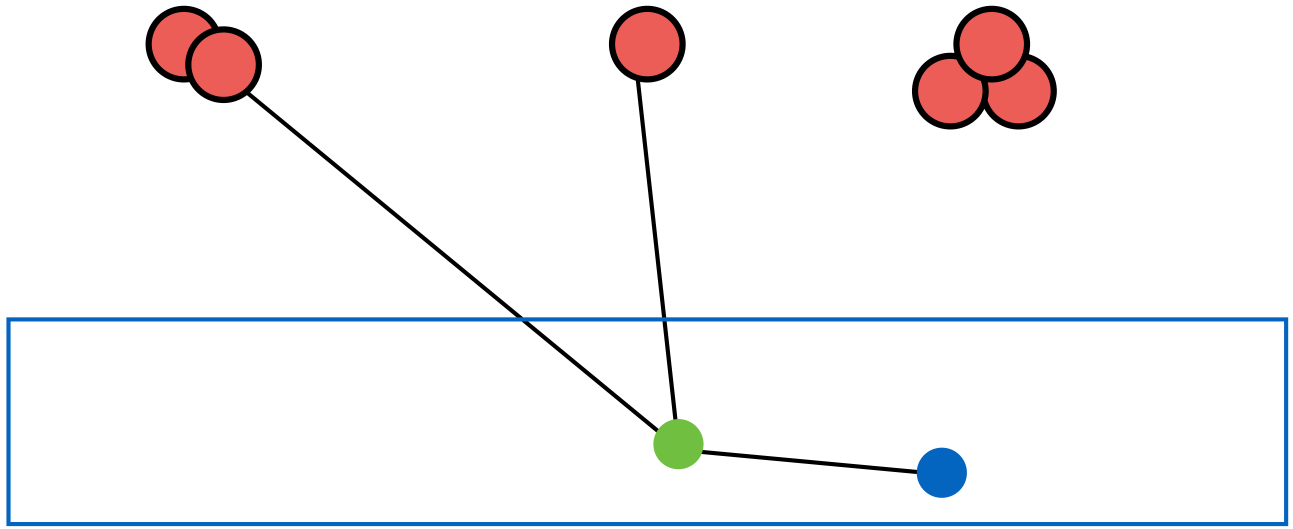

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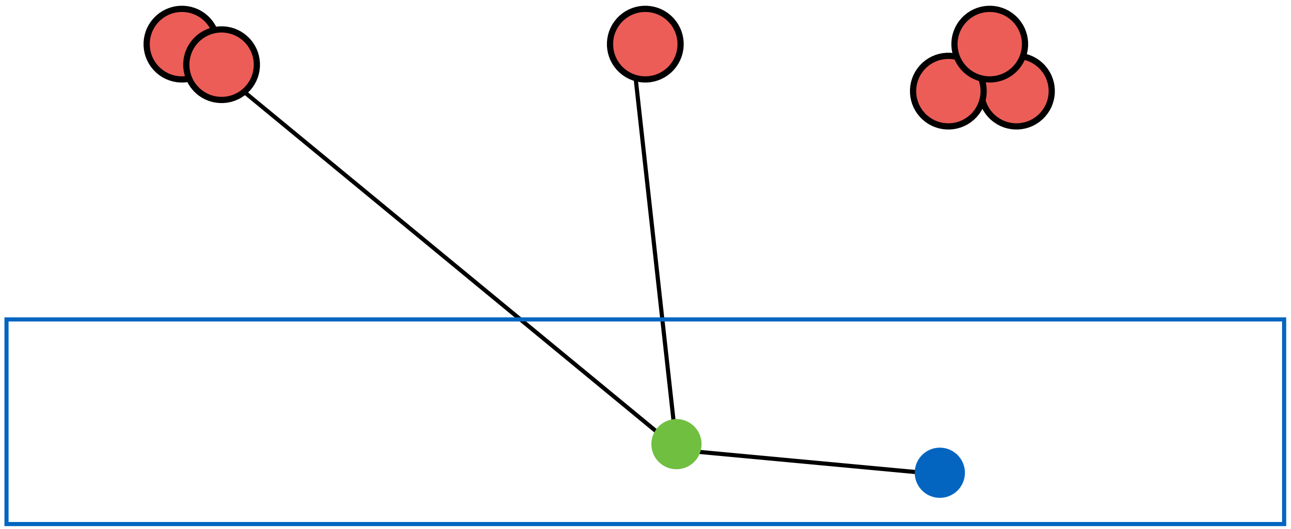

…a leaf with at least two neighbors in different components.

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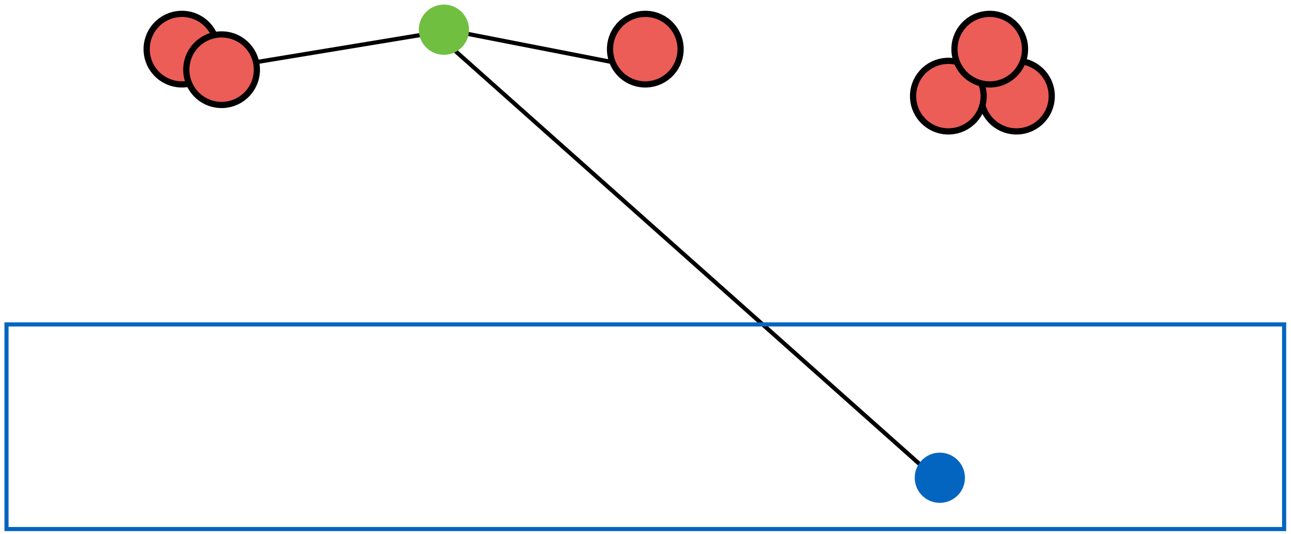

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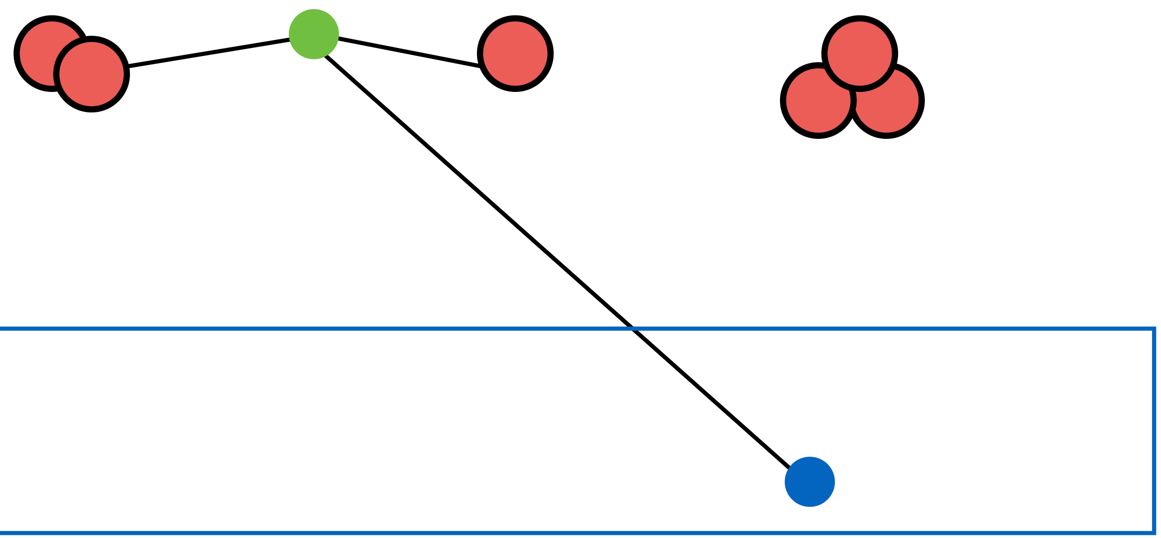

The leaf merges two components when we don't include it.

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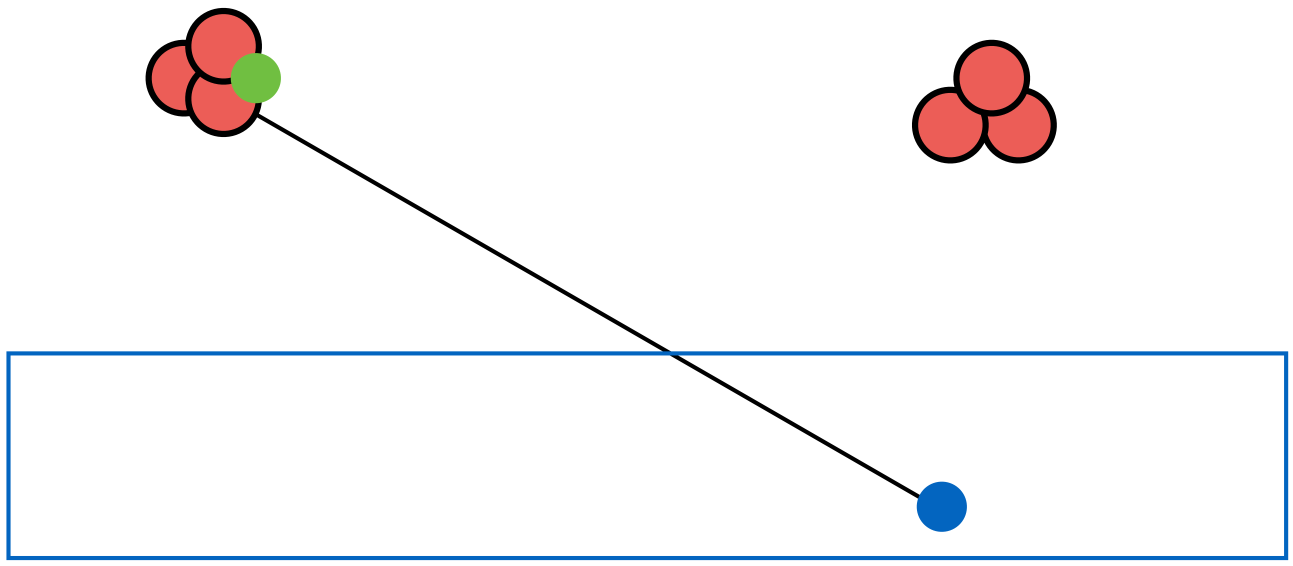

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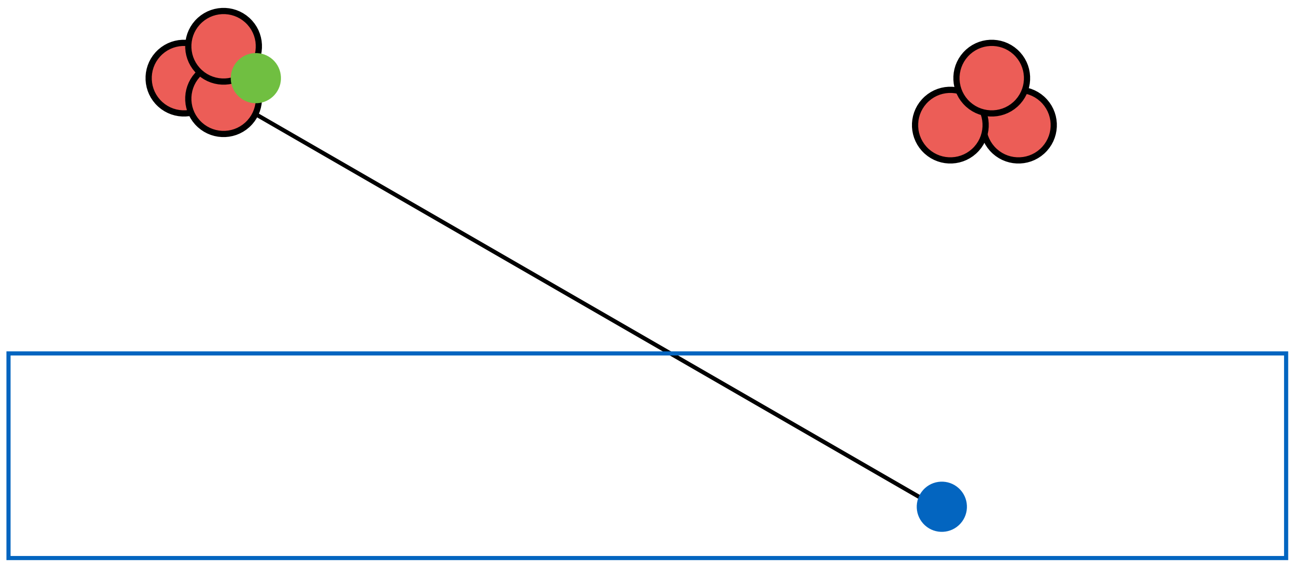

The number of components "on top" decreases.

Feedback Vertex Set

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A graph on n vertices, m edges and an integer k.

Is there a subset of at most k vertices whose removal makes the graph acyclic?

Question

Start with a leaf.

Two neighbors in one component: forced.
At most one neighbor above: preprocess.
At least two neighbors, all in different components above: branch.

Feedback Vertex Set

Input A graph on n vertices, m edges and an integer k.

Is there a subset of at most k vertices whose removal makes the graph acyclic? Question

FEEDBACK VERTEX SET

Input A graph on n vertices, m edges and an integer k.

Is there a subset of at most k vertices whose removal makes the graph acyclic? Question

Let t denote the number of components among the red vertices. Let $w = (k+t)$.

FEEDBACK VERTEX SET

Input A graph on n vertices, m edges and an integer k.

Is there a subset of at most k vertices whose removal makes the graph acyclic? **Question**

Let t denote the number of components among the red vertices. Let w = (k+t).

Include v… k drops by 1
Exclude v… t drops by at least 1

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Let t denote the number of components among the red vertices. Let w = (k+t).

Include v… k drops by 1
Exclude v… t drops by at least 1

Either way, w drops by at least one.

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Input A graph on n vertices, m edges and an integer k.

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Either way, w drops by at least one.

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Input A graph on n vertices, m edges and an integer k.

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Either way, w drops by at least one.

Running time: $2^w = 2^{(k+t)}$

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Either way, w drops by at least one.

Running time: $2^{w} = 2^{(k+t)} \leq 2^{k+k} \leq 4^{k}$

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Overall Running Time…

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Either way, w drops by at least one.

Running time: $2^w = 2^{(k+t)} \leq 2^{k+k} \leq 4^k$

Overall Running Time...

$$\sum_{i=1}^{k} \binom{k}{i} 4^k = 5^k$$

Feedback Vertex Set

Input A tournament T on n vertices and an integer k.

Is there a subset of k arcs that can be reversed to make the tournament acyclic? Question

This implies an $O(5^k)$ algorithm for FVS.

Feedback Vertex Set

Input

A tournament T on n vertices and an integer k.

Is there a subset of k arcs that can be reversed to make the tournament acyclic?

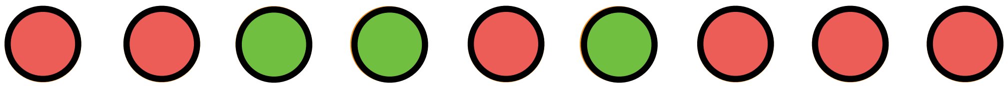
Question

This implies an $O(5^k)$ algorithm for FVS.

Feedback Vertex Set

Input A graph on n vertices, m edges and an integer k.

Is there a subset of at most k vertices whose removal makes the graph bipartite? **Question**



A Odd Cycle Transversal of size (k+1).

Let us "guess" how a OCT of size at most k interacts with this one.

Odd Cycle Transversal (OCT)

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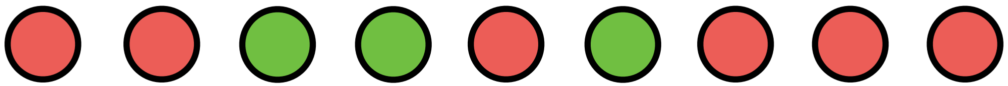
Is there a subset of at most k vertices whose removal makes the graph bipartite? **Question**



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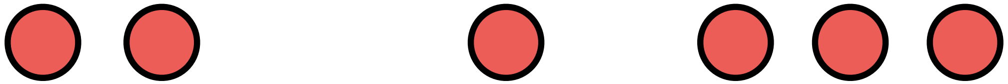

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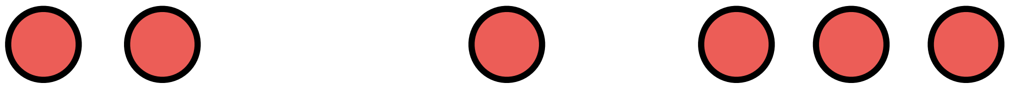

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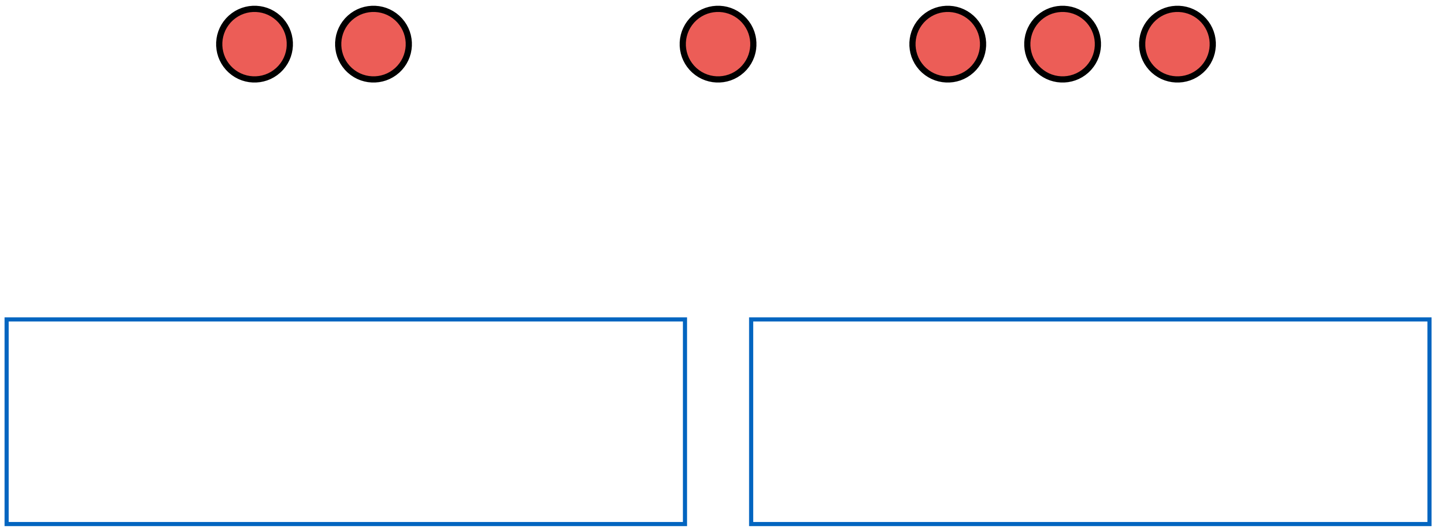

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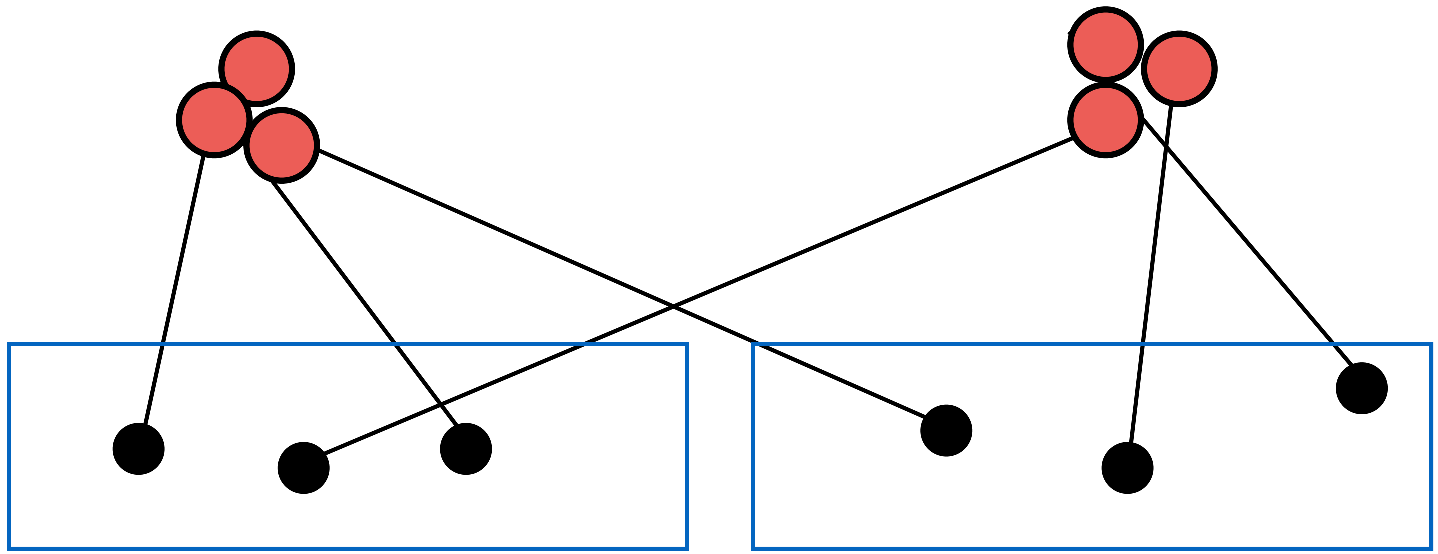

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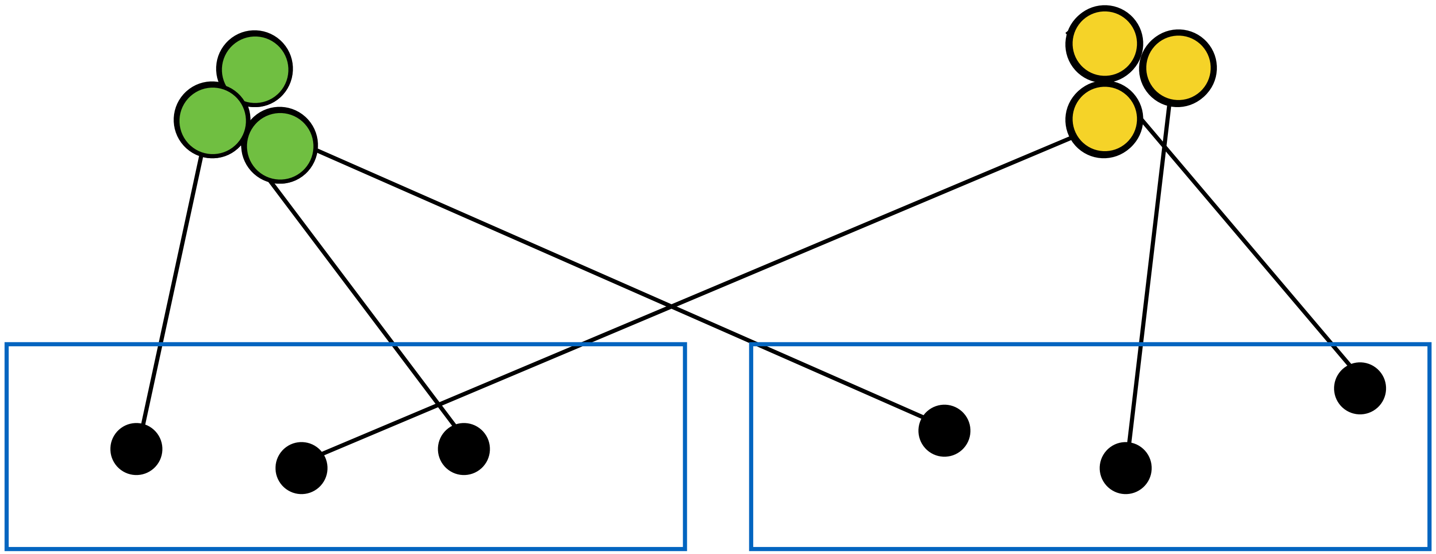

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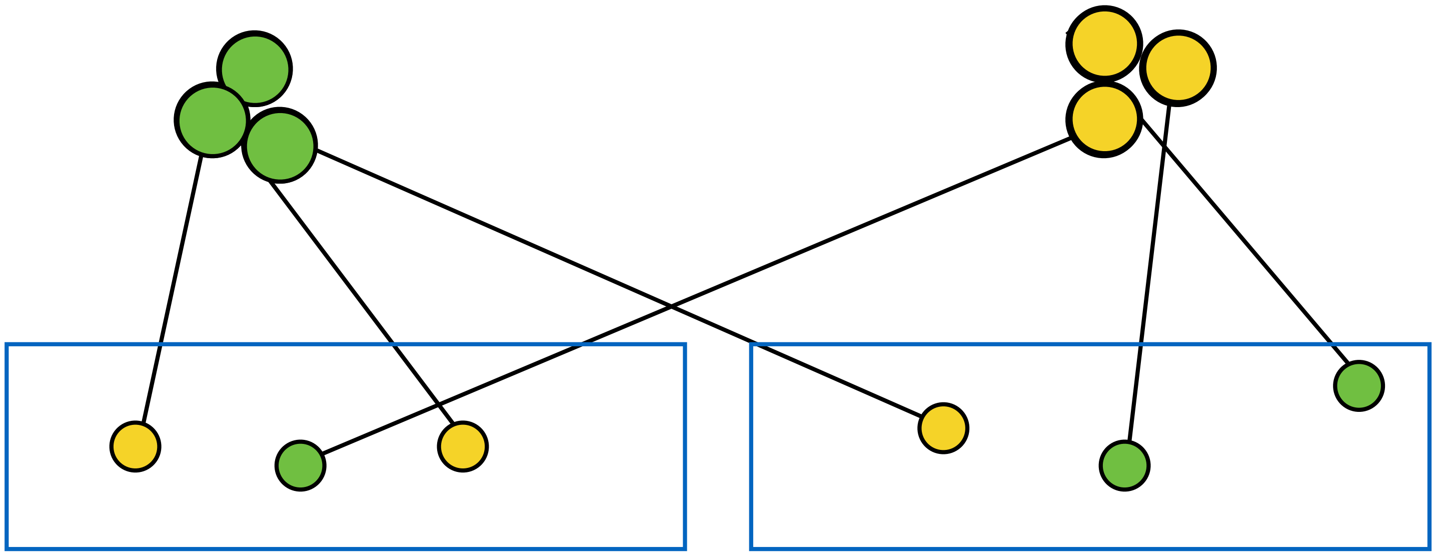

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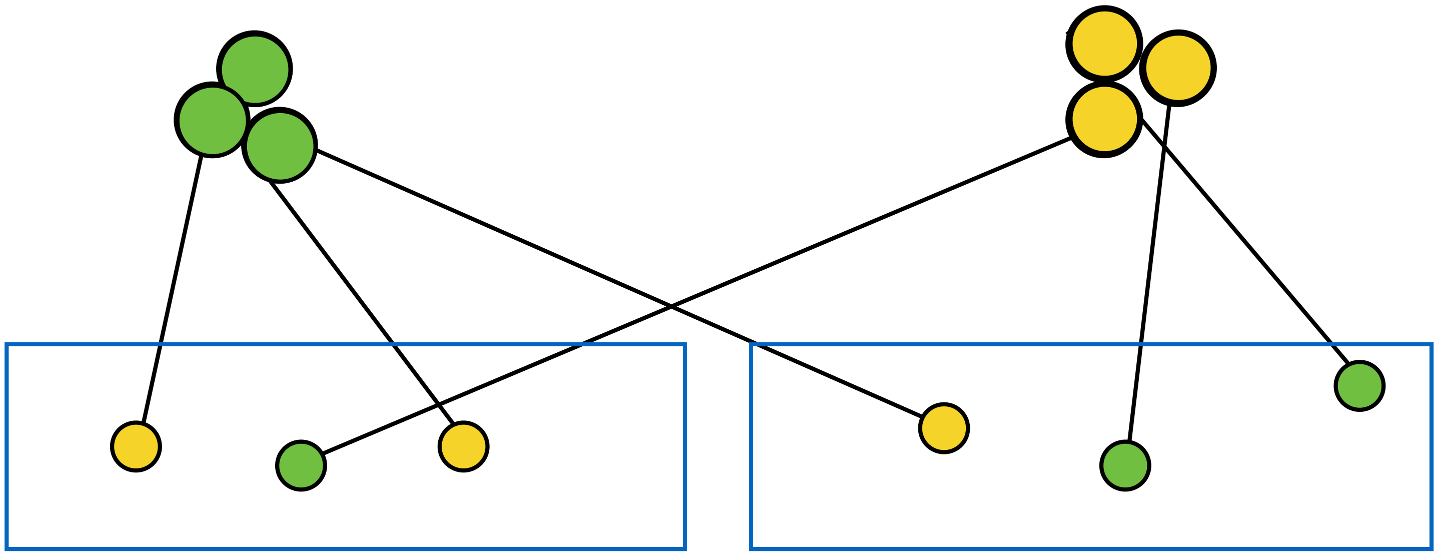
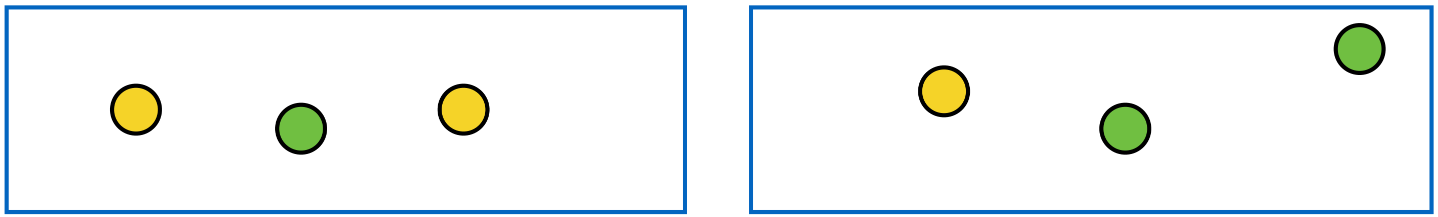


Odd Cycle Transversal (OCT)

Input A graph on n vertices, m edges and an integer k.

Is there a subset of at most k vertices whose removal makes the graph bipartite? **Question**



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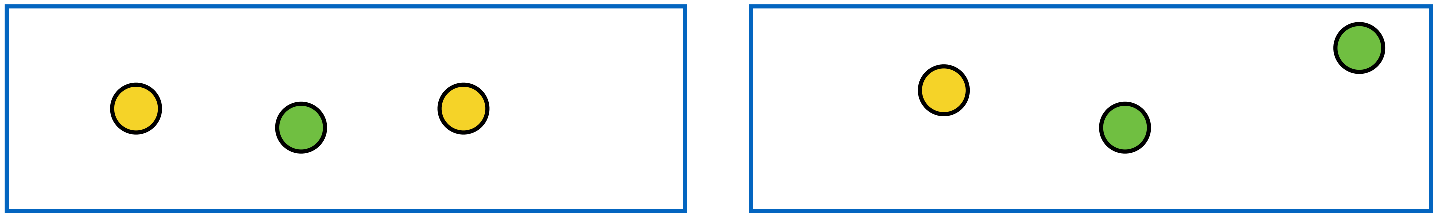

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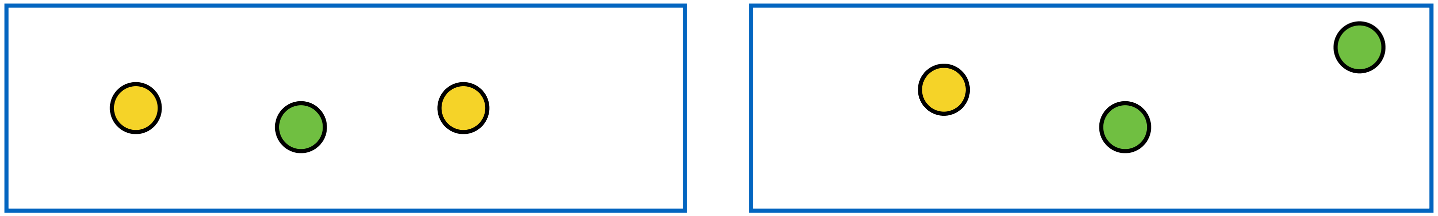

Can we remove at most b vertices so that the resulting graph has a bipartition extending this pre-coloring?

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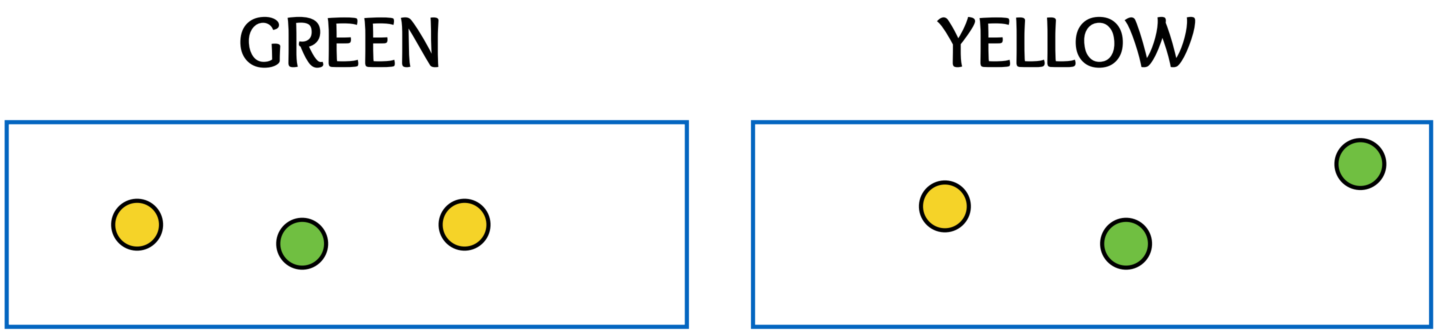

Can we remove at most b vertices
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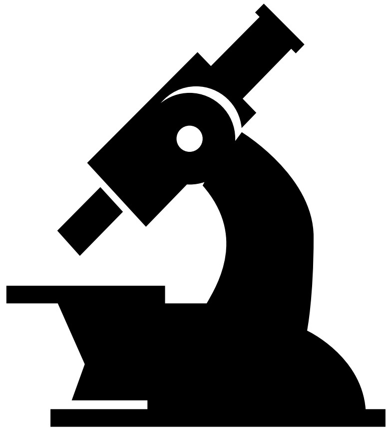
Odd Cycle Transversal (OCT)

Reading Assignment

- OCT parameterized by solution size k has a $O^*(3^k)$ FPT algorithm.
- Using the Iterative Compression framework, it would be enough to solve the Disjoint OCT problem in $O^*(2^k)$ time.
- This means that it would be enough to solve the pre-coloring preserving deletion to bipartition problem given in the previous slide in polynomial time! How can this be done?



Take Away...

Show that the problem has a hereditary property

Solve the "compression" question, or even a "Disjoint" version.

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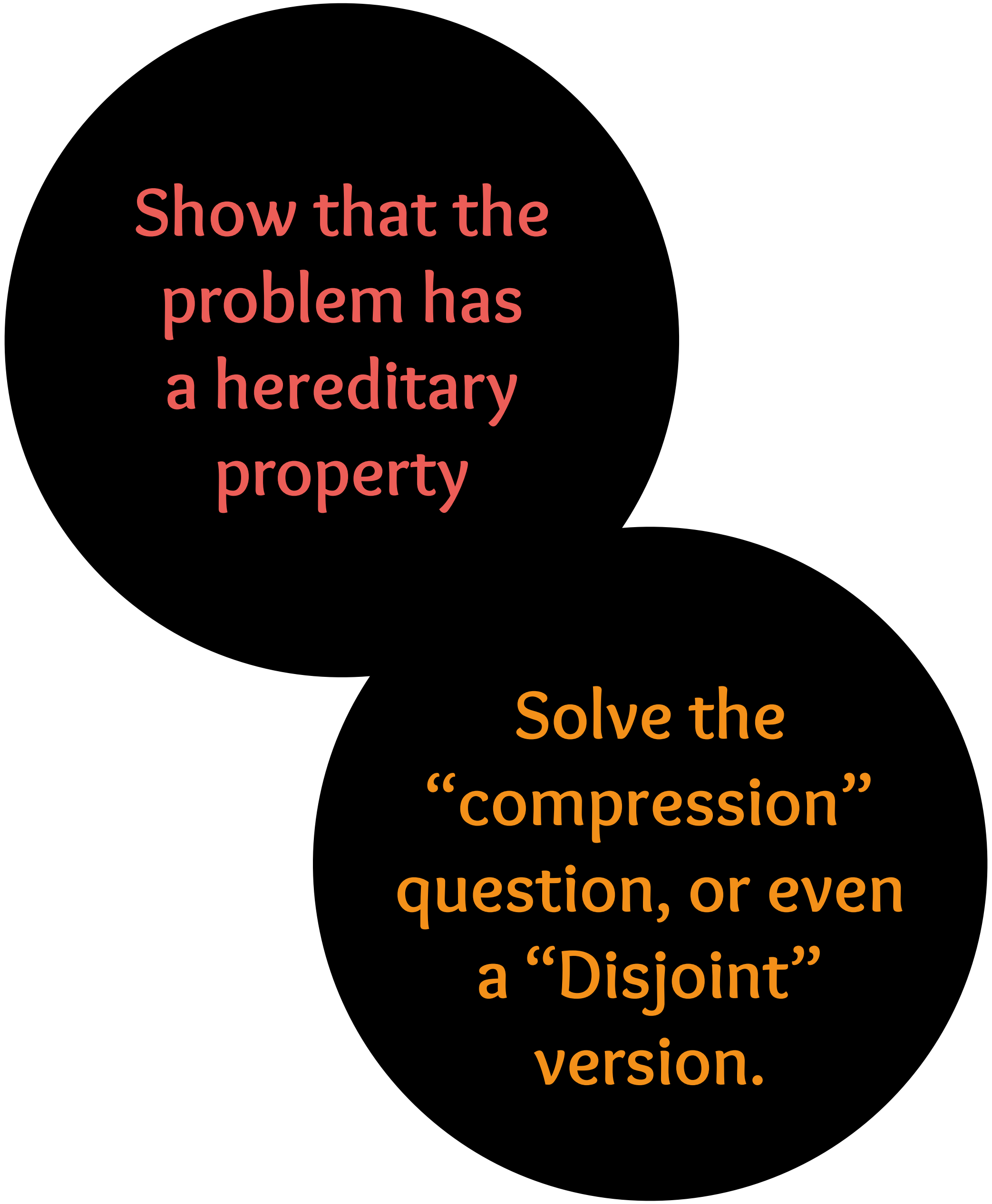

Is a framework that can be setup quite easily.

The compression algorithm can be fairly non-trivial.