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INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR  
CS21003 Algorithms I: End Semester Examination 2022 Spring

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Date of Examination: 12<sup>th</sup> April 2022

Duration: 55 minutes + 5 minutes (for scanning, concatenating, and uploading)

Full Marks: 20

Subject: CS21003 Algorithms I

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**Part 1**

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1. (a) Show that the average depth of a node in an  $n$ -node binary search tree is  $\mathcal{O}(\log n)$ .
- (b) We say that a string  $y$  has a repetition factor of  $r$  if  $r$  is the largest integer such that there exists a string  $x$  such that  $y$  is  $x$  concatenated with itself  $r$  times. For a pattern  $P[1, \dots, m]$ , we denote the repetition factor of  $P_i = P[1, \dots, i]$  by  $\rho(P_i)$ . Design a deterministic  $\mathcal{O}(m)$  time algorithm to compute  $\rho(P_1), \dots, \rho(P_i)$ .

[3 + 7 Marks]

2. (a) Consider the following problem. Input consists of an array  $\mathcal{A}[1, \dots, n]$  of distinct integers and an integer  $x$ . If  $x = \mathcal{A}[k]$  for some index  $k \in [n]$ , then output  $k$ ; otherwise output  $-1$ . Design a deterministic  $\mathcal{O}(n)$ -time algorithm for the above problem.

**The question is wrong. Actual question: If  $x = \mathcal{A}[k]$  for some index  $k \in [n]$ , then find the integer  $\ell$  such that  $x$  is the  $\ell$ -th smallest integer.**

**Good news: You get full marks in this question.**

- (b) Show the execution of Knuth-Morris-Pratt algorithm for string matching on text “abaabab” and pattern “abab.”

[7 + 3 Marks]

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*All the best!!!*

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