Tutorial 1: CS21003 Algorithms 1

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1. Find fallacy in the following derivation.

   Claim 1. $n = \mathcal{O}(1)$

   Proof. We will proof by induction on n. For $n = 0$, the statement is clearly true. Let us assume that the statement holds for $n = n_1$. We will now prove it for $n = n_1+1$. We have the following.

   $$n_1 + 1 = \mathcal{O}(n_1 + 1) = \mathcal{O}(n_1) = \mathcal{O}(1)$$

   The third equality follows from induction hypothesis. □

2. Arrange the following functions

   $$n, 10, n^{\ln n}, \ln \ln^2 n, (\ln n)^{\ln n}, 2^{\sqrt{n}}, n!, n \ln^{1000} n, n^{1.0001}, n^{.999} \ln^{100000} n, \sqrt{n}^{\sqrt{n}}, \frac{1}{n}, \binom{n}{\frac{n}{2}}$$

   in a sequence such that if $f_1(n)$ appears in the left of $f_2(n)$, then $f_1(n) = \mathcal{O}(f_2(n))$.

3. Develop recursive definitions to solve the following problems and refine them for final algorithms and data structures.

   (a) Given two unordered sets U and V, find their union, intersection and set difference.

   (b) Given two ordered sets X and Y of integers, find their union, intersection and set difference.

4. Compute asymptotic complexity of $T(n, n)$ in terms of Θ where

   $$T(x, y) = \begin{cases} x & \text{if } y \leqslant 50 \\ y & \text{if } x \leqslant 50 \\ x + y + T\left(\frac{2x}{3}, \frac{y}{2}\right) & \text{otherwise} \end{cases}$$

5. Compute asymptotic complexity of $T(n)$ in terms of Θ where

   (a) $T(n) = \begin{cases} 3T(\lceil n/2 \rceil) + n \log_2 n & \text{if } n > 50 \\ 1 & \text{otherwise} \end{cases}$

   (b) $T(n) = \begin{cases} T(\lceil n/5 \rceil) + 9n & \text{if } n \geqslant 50 \\ 1 & \text{otherwise} \end{cases}$

   (c) $T(n) = \begin{cases} T\left(\lceil \frac{n}{2} \rceil + 7\right) + \left(\lfloor \frac{n}{2} \rfloor + 11\right) & \text{if } n \geqslant 50 \\ 1 & \text{otherwise} \end{cases}$

   (d) $T(n) = \begin{cases} T(\lceil \sqrt{n} \rceil) + 13 \lg n & \text{if } n \geqslant 50 \\ 1 & \text{otherwise} \end{cases}$

6. Give an example of two functions $f, g : \mathbb{N} \longrightarrow \mathbb{N} \setminus \{0\}$ such that we have neither $f = \mathcal{O}(g)$ nor $f = \Omega(g)$.