Algorithmic Game Theory
Practice Problems: Mechanism Design, Stable Matching

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1. (Inspired by an exercise from [Nar14]) Consider a scenario with a set $[5]$ five sellers selling identical items with valuations $v_1 = 23, v_2 = 15, v_3 = 11, v_4 = 8, v_5 = 2$ and one buyer. Compute VCG payments in each of the following cases.
   (i) The buyer wishes to buy 3 items and each seller can supply at most one items.
   (ii) The buyer wishes to buy 3 items and each seller can sell at most 2 items.
   (iii) The buyer wishes to buy 6 items and each seller can sell at most 2 items.
2. Prove that in a selfish load balancing game with 3 tasks and 2 identical machines, the PoA with respect to PSNE is 1.
3. Consider a stable matching instance with a set $\mathcal{A}$ of n men and another set $\mathcal{B}$ of n women. For each woman $w \in \mathcal{B}$, we define $h(w)$ to be the least preferred man $m \in \mathcal{A}$ by the woman w with whom she can be matched in some stable matching. A matching is called women-pessimal if every woman $w \in \mathcal{B}$ is matched with $h(w)$. Prove that the stable matching output by the men-proposal deferred acceptance algorithm is women-pessimal.
4. In a stable matching instance with sets $\mathcal{A}$ and $\mathcal{B}$ of n men and women, suppose $\mathcal{M}_1$ and $\mathcal{M}_2$ be two stable matchings. Define $\mathcal{M}_3 = \{(a, b) \in \mathcal{A} \times \mathcal{B} : \mathcal{M}_1(a) = \mathcal{M}_2(a) = b \text{ or } b = \mathcal{M}_1(a) \succ_a \mathcal{M}_2(a) \text{ or } b = \mathcal{M}_2(a) \succ_a \mathcal{M}_1(a)\}$; that is, in $\mathcal{M}_3$, every man $a \in \mathcal{A}$ gets his better partner between $\mathcal{M}_1(a)$ and $\mathcal{M}_2(a)$. Prove that $\mathcal{M}_3$ is also a stable matching.

References

[Nar14] Y. Narahari. *Game Theory and Mechanism Design*. World Scientific Publishing Company Pte. Limited, 2014.