Assignment 1: CS21003 Algorithms 1

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1. Compute asymptotic complexity of $T(n)$ in terms of Θ for the following recurrence.

$$T(n) = \begin{cases} T\left(\lceil \frac{n}{3} \rceil\right) + T\left(\lceil \frac{n}{7} \rceil\right) + 11n + \lceil \sqrt{n} \rceil & \text{if } n \geqslant 37 \\ 37^{37} & \text{otherwise} \end{cases}$$

[5 Marks]

2. Prove or disprove: (i) $f(n) = \Theta\left(f\left(\frac{n}{2}\right)\right)$ (ii) $f(n) + g(n) = \Theta(\min(f(n), g(n)))$

[2.5+2.5 Marks]

3. Let there be n houses located on a 2-D plane, each denoted by the coordinate location (x,y) of their electrical poles. There is one power generation source, P also having its coordinates (p,q) and can generate enough power to serve all the houses. The cost of electrically connecting two poles is equal to the distance between the poles. We may assume that each connection can transmit sufficient power. You are to connect the houses electrically in such a manner that every house has an electricity connection that eventually connects with the power generation source, either by a direct connection or indirectly through other houses. The problem is to find the minimum cost connection for all the houses. You are to do the following:

   (a) Develop a recursive definition for the problem.
   (b) Prove the correctness of your algorithm
   (c) Analyze the complexity of your algorithm based on (a)
   (d) Develop the recursion tree and present its properties
   (e) Choose the best option from your choice points, justifying your choice
   (f) Decide on the data structures
   (g) Present the final algorithm
   (h) Analyze its complexity
   (i) Show the working on (a), (d) and (h) on a non-trivial example of 10 houses and one power generation station
   (j) How would the algorithm change if there were k power generation stations, each generating sufficient power?

[10 Marks]