INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
CS21003 Algorithms I: Second Class Test 2021 Spring

Date of Examination: 20th February 2021
Duration: 35 minutes + 5 minutes (for scanning, concatenating, and uploading)
Full Marks: 10
Subject: CS21003 Algorithms I

Part I

1. Consider the following variation on the Interval Scheduling Problem. You have a processor that can operate 24 hours a day, every day. People submit requests to run daily jobs on the processor. Each such job comes with a start time and an end time; if the job is accepted to run on the processor, it must run continuously, every day, for the period between its start and end times. (Note that certain jobs can begin before midnight and end after midnight; this makes for a type of situation different from what we saw in the Interval Scheduling Problem.)

   Given a list of n such jobs, your goal is to accept as many jobs as possible (regardless of their length), subject to the constraint that the processor can run at most one job at any given point in time. Provide an algorithm to do this with a running time that is polynomial in n. You may assume for simplicity that no two jobs have the same start or end times.

   Example. Consider the following four jobs, specified by (start-time, end-time) pairs.

   (6 P.M. , 6 A.M.), (9 P.M. , 4 A.M.), (3 A.M. , 2 P.M.), (1 P.M. , 7 P.M.).

   The optimal solution would be to pick the two jobs (9 P.M. , 4 A.M.) and (1 P.M., 7 P.M.), which can be scheduled without overlapping.

   (a) Design a greedy algorithm for the above problem. (Hint: A modified version of the greedy algorithm for the activity selection problem works)
   (b) Prove correctness of your algorithm.
   (c) Analyse the complexity of your algorithm.
   (d) Show the working of your algorithm on the following example.
   (6 P.M. , 6 A.M.), (9 P.M. , 4 A.M.), (3 A.M. , 2 P.M.), (1 P.M. , 7 P.M.).

[3+3+2+2 Marks]

All the best
