INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Algorithmic Game Theory 2021-22: Sample Solution Sketch of Fourth Class Test

Part 1

Let d be the last digit (from right) of your roll number. Suppose we have 3 buyers who want to buy 2 goods A and B. Each good can be allocated to any buyer independent of the other good. Their valuations are as follows.

Bundle	v_1	v_2	v_3
$\emptyset$	0	0	0
$\{A\}$	$d+1$	d	$d+1$
$\{B\}$	$d+2$	$d+3$	$d+2$
$\{A, B\}$	$d+3$	$d+3$	$d+4$

[10 Marks]

Suppose we have $d = 5$. Then we have the following valuations. An allocatively efficient allocation

Bundle	v_1	v_2	v_3
$\emptyset$	0	0	0
$\{A\}$	6	5	6
$\{B\}$	7	8	7
$\{A, B\}$	8	8	9

is to give A to player 1 and B to player 2. The VCG payment of the players is as follows.
Payment of player 1 $= (-6-8) - (-6-8) = 0$.
Payment of player 2 $= (-6-8) - (-6-7) = -1$.
Payment of player 1 $= (-6-8) - (-6-8) = 0$.

Part 2

Prove or disprove:

1. The number of iterations that the men proposing deferred acceptance algorithm takes on an instance of the stable marriage problem is independent of how/which unmatched man we pick in an iteration.

   We observe that the number of iterations in the men-proposing deferred acceptance algorithm is the total number of rejections plus the number of men. Since the output of the men-proposing deferred acceptance algorithm is the men-optimal stable matching, the number of rejections in any run of the men-proposing deferred acceptance algorithm is the same.

2. The output of the men-proposing deferred acceptance algorithm is woman-pessimal. That is, there does not exist any other stable matching where a woman is matched with a man whom she prefers less than her partner in the output of the men-proposing deferred acceptance algorithm.

   Similar to the way we proved that the output of the men-proposing deferred acceptance algorithm is men-optimal.

[5+5 Marks]