INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Algorithmic Game Theory 2021-22: Sample Solution Sketch of Second Class Test

The last digit of your roll number from right be d.

$$A = \begin{bmatrix} d+2 & d+3 \\ d+4 & d+1 \end{bmatrix}$$

Write the row and column players' linear programs for the above matrix game given by A. Also compute all MSNEs of the above matrix game.

[5+5 Marks]

Row player's linear program is the following.

maximize t

subject to:

$$t \leqslant (d+2)x_1 + (d+4)x_2$$
$$t \leqslant (d+3)x_1 + (d+1)x_2$$
$$x_1 + x_2 = 1$$
$$x_1 \geqslant 0, x_2 \geqslant 0$$

Column player's linear program is the following.

minimize w

subject to:

$$w \geqslant (d+2)y_1 + (d+3)y_2$$
$$w \geqslant (d+4)y_1 + (d+1)y_2$$
$$y_1 + y_2 = 1$$
$$y_1 \geqslant 0, y_2 \geqslant 0$$

Solving the row player's linear program, we obtain $x_1 = 0.75, x_2 = 0.25$ and solving the column player's linear program, we obtain $y_1 = 0.5, y_2 = 0.5$. Hence, the unique MSNE of the above matrix game is $(\{1:0.75, 2:0.25\}, \{1:0.5, 2:0.5\})$.

In Local-Weighted-Max-2SAT problem, we are given a set of 2SAT clauses each having a weight. An assignment of the variables is said to satisfy a clause if and only if it makes at least one of its literal true. The goal is to find an assignment which is locally optimal — by changing the assignment of any one variable, it is not possible to increase the sum of weights of the clauses satisfied. Prove that Local-Weighted-Max-2SAT is PLS-complete.

[10 Marks]

The Local-Weighted-Max-2SAT clearly belongs to the complexity class PLS as it can be modeled as an abstract search problem. To show PLS-hardness, we reduce from Local-Max-Cut. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, w : \mathcal{W} \longrightarrow \mathbb{R}_{>0})$ be an arbitrary instance of Local-Max-Cut. We consider the following instance of Local-Weighted-Max-2SAT.

The set of variables: $x_v$for all $v \in \mathcal{V}$

The set of clauses: for all edge $\{u, v\} \in \mathcal{E}$, $(x_u \vee x_v)$ and $(\overline{x_u} \vee \overline{x_v})$ of weight $w(\{u, v\})$

For a cut $(\mathcal{S}, \mathcal{V} \setminus \mathcal{S})$ of $\mathcal{G}$ of weight $w(\mathcal{S}, \mathcal{V} \setminus \mathcal{S})$, assigning the corresponding variables in $\mathcal{S}$ to TRUE and others to FALSE satisfies clauses of weight $\sum_{e \in \mathcal{E}} w(e) + w(\mathcal{S}, \mathcal{V} \setminus \mathcal{S})$. Clearly, the cuts in $\mathcal{G}$ and the assignments of the variables are in one-to-one correspondence. In particular, local maximum of $\mathcal{G}$ are also in one-to-one correspondence with the local optimal solutions of the above Local-Weighted-Max-2SAT instance.