
INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Parameterized Algorithms: Class Test 3 2020-21

Date of Examination: 22 October 2020
Duration: 35 Minutes + 10 Minutes (for uploading answer scripts on Moodle)
Full Marks: 15
Subject No: CS60083
Subject: Parameterized Algorithms
Department/Center/School: COMPUTER SCIENCE AND ENGINEERING

You may refer to the book and all lecture slides during the exam. Please cite results that you are not proving in your answer script.

1. The *distance* $d^{\mathcal{G}}(v, w)$ between two vertices v, w of a graph $\mathcal{G}$ is the minimum of the lengths of paths from v to w, or ∞ if no such path exists. Let $k \in \mathbb{N}$ and $\mathcal{G} = (V, E)$ be a graph. An $L(2, 1)$-*k-coloring* of $\mathcal{G}$ is a mapping $C : V \to [k]$ such that for all $v, w \in V$:

   ▷ If $d^{\mathcal{G}}(v, w) = 1$, then $|C(v) - C(w)| \geqslant 2$.
   ▷ If $d^{\mathcal{G}}(v, w) = 2$, then $|C(v) - C(w)| \geqslant 1$.

   Such colorings are motivated by the problem of assigning frequencies to transmitters. Prove that for every $k \in \mathbb{N}$ the following problem is fixed-parameter tractable:

p^*-*tw*-$L(2, 1)$-COLORING
Instance: A graph $\mathcal{G}$ and $k \in \mathbb{N}$.
Parameter: $tw(\mathcal{G}) + k$.
Problem: Decide whether $\mathcal{G}$ has an $L(2, 1)$-*k-coloring*.

   Hint: Use Courcelle's Theorem

   [10 Marks]

2. A graph $\mathcal{G}$ is said to be a k-IC-graph if $V(\mathcal{G})$ can be partitioned into k parts such that each part is either an independent set or a clique. Assuming $k \geqslant 3$, recognising whether a graph is a k-IC-graph is NP-complete. Given a $k \geqslant 3$, design a $2^n n^{O(1)}$ exact algorithm for recognising if a given graph is a k-IC-graph or not.

   [5 Marks]

Best of luck